More Quality Criteria, Fewer Bidders in Scoring Auctions

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Scoring auctions: the usual margin, and ours

Bids are ranked by a score $s = \alpha\, q - b$: quality q, price b, weight on quality α. The literature varies α.

Two tenders, both $\alpha = 0.7/0.3$ (70 quality, 30 price points of 100):

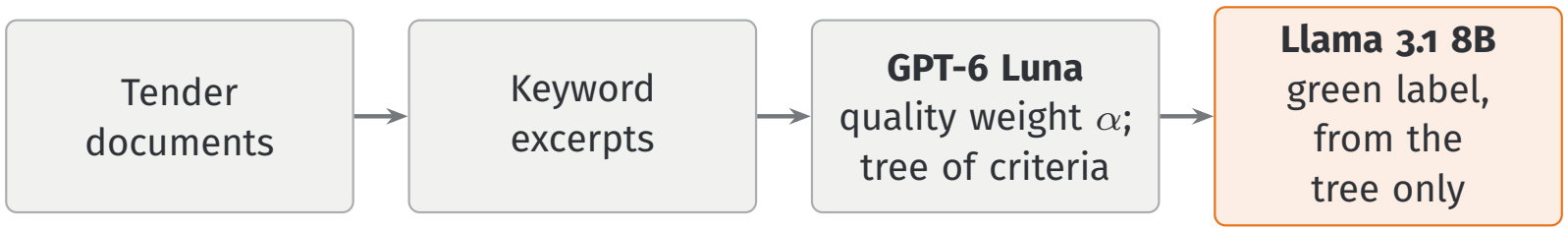
Tender A: $K = 2$	
staffing	40
service organisation	30

Tender B: $K = 5$	
staffing	25
service organisation	20
eco-labelled detergents	10
waste management	8
staff training	7

Question: not varying α, but varying K.

Data

- Italian cleaning-service tenders, **2012–2021**: bidders, subcontracting, reserve price, buyer (ANAC).
- About 56,000 tender documents, cut to keyword windows around the scoring rules.



- Regressions: **794 tenders, 184 buyers**. Mean 12 bidders, 49% green, quality weight 65 points.
- Robustness labels on the same trees: GPT-4.1 mini, GPT-6 Luna, GPT-6.1 Sol.

Sample: 1,974 tenders $\rightarrow$ 1,319 with a stated weight $\rightarrow$ 1,227 with a tree $\rightarrow$ 794 in the regressions. Complexity = rows of the tree (mean 12.4).

Green label: an example

Tree (GPT-6 Luna)

- Increased service frequency
  - Daily cleaning increased from twice to three times for specified items
  - Weekly cleaning increased from once to twice
  - Cleaning increased from monthly to every 20 consecutive days
- Software provision
- Use of eco-friendly cleaning products

Excerpt (Italian, as the models read it)

[…] sulla base dei seguenti criteri: id criterio descrizione punteggio massimo per ciascun criterio 1 aumento della frequenza del servizio 32 2 software 6 3 utilizzo di prodotti di pulizia ecologici 2 punteggio tecnico massimo (pt) 40 […]

8 punti se l'impresa propone un incremento da due volte al giorno a tre volte al giorno. […]

verranno attribuiti 2 punti nel caso in cui l'impresa si impegni ad utilizzare esclusivamente prodotti dotati del marchio ambientale ecolabel o equivalente. […]

Llama 3.1 8B: green = 1

environmental row: *use of eco-friendly cleaning products*

Tender in Rome, 2014, technical weight 40/100: a separate green criterion worth 2 points (Ecolabel-only products). Every label: green.

Regression: number of bidders on green

$$\text{bidders}_i = \beta\, \text{green}_i + \gamma' x_i + \text{year FE} + \text{buyer FE} + \varepsilon_i$$

Green label	Precision	Recall	F1	$\hat{\beta}$
Llama 3.1 8B (baseline)	0.99	0.63	0.77	-4.48^{***}
GPT-4.1 mini	0.99	0.70	0.82	-4.29^{***}
GPT-6 Luna	0.97	0.80	0.88	-3.30^{***}
GPT-6.1 Sol	0.97	0.82	0.89	-4.02^{***}

- **Green coefficient** β: same buyer, same year, a tender with a green criterion has **about 4.5 fewer bidders** (mean 12).
- **Validation sample**: 300 random procedures coded by hand, blind to the labels. Precision near 1: almost no false greens. Stronger models find more green (recall), and $\hat{\beta}$ stays between -3.3 and -4.5.

794 tenders, 184 buyers; x: log reserve price, open procedure; robust SEs. Validation: 222 of the 300 have a tree, 64% green; coded by Claude under written rules.

A puzzle

Green criteria go with **about 4.5 fewer bidders**, within buyer.

Why does adding green reduce entry?

The data cannot say. We need a model.

Model, part 1: no entry costs

The scoring auction

Model

N risk-neutral bidders, private efficiency $\theta_i \in [\underline{\theta}, \bar{\theta}]$, i.i.d. F. Bidder i offers a price b_i and qualities $q_{i1}, \ldots, q_{iK}$; the score is

$$s_i = \frac{\alpha}{K} \sum_{j=1}^{K} q_{ij} - b_i,$$

and the winner earns $b_i - \sum_j C(q_{ij}, \theta_i)$, with C increasing and strictly convex in q, decreasing in θ, $C(0, \theta) = 0$, and $C_{q\theta} \leq 0$: better types have lower marginal cost.

- A design is (α, K): total quality weight α, shared by K criteria.
- We vary **one at a time**: first α, then K.

Symmetric criteria: one average quality

Lemma 1 (equal quality on every criterion)

A bidder sets $q_j = Q$ for all j. In the average quality Q:

$$s = \alpha Q - b, \qquad \text{quality cost} = K\, C(Q, \theta).$$

Derivation: for any target score the bidder maximises

$$\frac{\alpha}{K} \sum_j q_j - \sum_j C(q_j, \theta) \quad \Longrightarrow \quad \frac{\alpha}{K} = C_q(q_j, \theta) \ \text{ for every } j,$$

and strict convexity gives the same q_j on every criterion.

Pseudo-types

Definition

The pseudo-type of type θ is its best score surplus from quality:

$$\rho_{\alpha,K}(\theta) = \max_{Q} \left\{ \alpha Q - K\, C(Q, \theta) \right\}.$$

With $\rho_{\alpha,K}$ increasing, the scoring auction is a one-dimensional auction in pseudo-types (Asker and Cantillon 2008): the bidder with the highest pseudo-type wins.

Warm-up: quadratic costs

Result

$$C(q,\theta) = \frac{q^2}{2\theta} \qquad \Longrightarrow \qquad \rho_{\alpha,K}(\theta) = \frac{\alpha^2}{2K}\,\theta$$

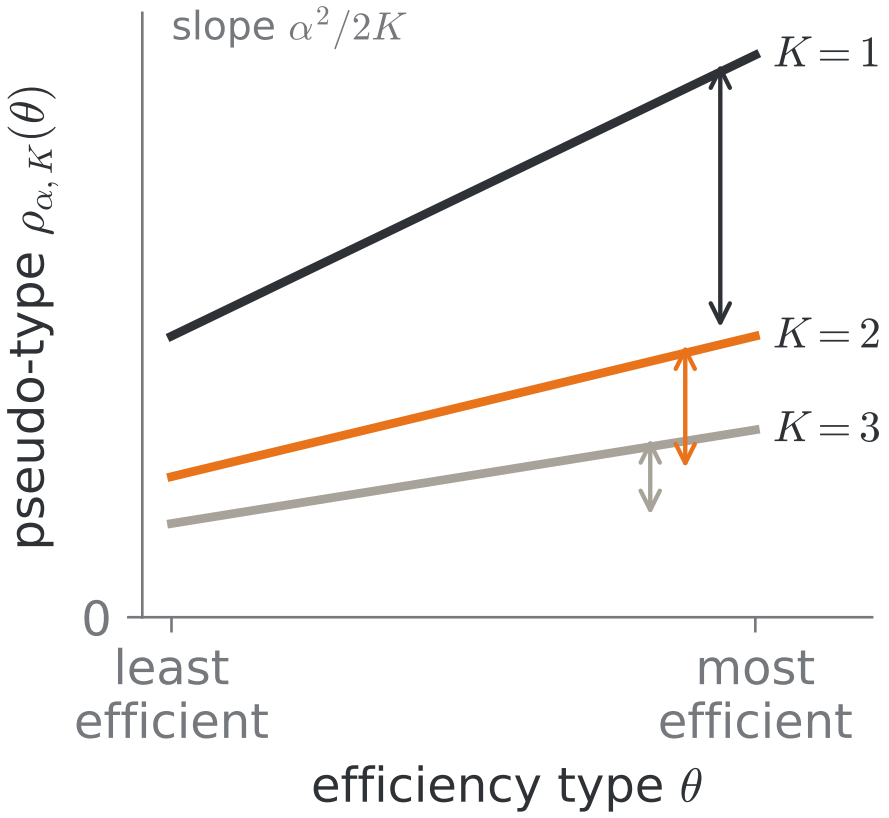
objective: $\max_{Q} \; \alpha Q - K\,\frac{Q^2}{2\theta}$

FOC: $\alpha = \frac{K\,Q}{\theta} \quad \Rightarrow \quad Q^*_{\alpha,K} = \frac{\alpha\,\theta}{K}$

plug in: $\rho_{\alpha,K} = \alpha\,\frac{\alpha\theta}{K} - \frac{K}{2\theta}\left(\frac{\alpha\theta}{K}\right)^2 = \frac{\alpha^2\theta}{K} - \frac{\alpha^2\theta}{2K} = \frac{\alpha^2}{2K}\,\theta$

- Slope $\alpha^2/(2K)$ falls in K: pseudo-types **compress**.
- Total effort $K\,Q^*_{\alpha,K} = \alpha\theta$ is fixed: the **same effort, spread thinner**.

Warm-up: pseudo-types in a picture



Slope $\alpha^2/(2K)$: each extra criterion narrows the gap between the most and the least efficient type (arrows).

Warm-up: rents

Result

$$\pi_{\alpha,K}(\theta) = \frac{\alpha^2}{2K} \int_{\underline{\theta}}^{\theta} F^{N-1}(x)\, dx,$$

decreasing in K for every θ.

Derivation: revenue equivalence gives $\pi_{\alpha,K}(\theta) = \int_{\underline{\theta}}^{\theta} F^{N-1} \rho'_{\alpha,K}$; put in the slope $\rho'_{\alpha,K} = \alpha^2/(2K)$.

Rents scale with $\alpha^2/(2K)$: up in α, down in K.

Optimal quality

Lemma 2 (quality per criterion)

The optimal quality $Q^*_{\alpha,K}$ depends on α and K only through α/K: it **rises in** α and **falls in** K.

Proof. The first-order condition of $\max_Q\{\alpha Q - K\,C(Q,\theta)\}$ is

$$C_q(Q^*_{\alpha,K},\theta) = \frac{\alpha}{K},$$

and C_q is increasing in q (strict convexity), so $Q^*_{\alpha,K}$ rises in α/K.

Corner: $Q^*_{\alpha,K} = 0$ when $\alpha/K \le C_q(0,\theta)$.

Pseudo-type compression and decompression

Definition

Take two designs, e.g. two pairs (α, K). Relative to design A, design B

$$\textbf{compresses} \text{ pseudo-types if } \quad \rho'_B(\theta) \leq \rho'_A(\theta) \text{ for every } \theta,$$
$$\textbf{decompresses} \text{ pseudo-types if } \quad \rho'_B(\theta) \geq \rho'_A(\theta) \text{ for every } \theta.$$

- Compression: strong firms stay strong, but their **lead in score surplus shrinks**.
- Rents $\pi(\theta) = \int F^{N-1}\rho'$ follow the slope: lower under compression, higher under decompression.

More weight: higher rents

Proposition 1

For $\alpha' > \alpha$, (α', K) decompresses pseudo-types relative to (α, K): $\rho'_{\alpha',K} \geq \rho'_{\alpha,K}$. So $\pi_{\alpha,K}(\theta)$ **rises in** α for every θ.

Proof. Envelope theorem in θ, then $C_{q\theta} \leq 0$ and Lemma 2:

$$\rho'_{\alpha,K} = -K\, C_\theta(Q^*_{\alpha,K}, \theta), \qquad \frac{\partial \rho'_{\alpha,K}}{\partial \alpha} = -K\, C_{q\theta}\, \frac{\partial Q^*_{\alpha,K}}{\partial \alpha} \geq 0.$$

Revenue equivalence: rents depend only on the slope,

$$\pi_{\alpha,K}(\theta) = \int_{\underline{\theta}}^{\theta} F^{N-1}(x)\, \rho'_{\alpha,K}(x)\, dx.$$

No curvature condition: more weight raises effort on every criterion, where efficiency differences matter more.

General costs: when do pseudo-types compress?

Proposition 2

If $C(q, \theta) = c(q)/\theta$ with c increasing and strictly convex and $\log c$ **weakly concave**, the slope $\rho'_{\alpha,K}$ falls in K: pseudo-types compress. So $\pi_{\alpha,K}(\theta)$ **falls in** K for every θ.

Idea: effort per criterion depends on α/K. More criteria move each criterion to a lower scale, where efficiency differences matter less.

Covers all power costs $c(q) = q^\gamma$, $\gamma > 1$. A very convex technology can weaken or reverse the effect.

Proof of Proposition 2 (1/4): a bid is a score

A type-θ bidder that wins with score $s = \alpha Q - b$ earns

$$b - K\,C(Q,\theta) = \underbrace{\alpha Q - K\,C(Q,\theta)}_{\text{maximised by } Q^*_{\alpha,K}} - s.$$

- Q is chosen apart from s: the bidder earns $\rho - s$, with $\rho = \rho_{\alpha,K}(\theta)$.
- Rivals bid $\sigma_{\alpha,K}(\rho)$, increasing; $H_{\alpha,K}$ is the cdf of ρ.

The bidder's problem is a one-dimensional first-price auction:

$$\pi_{\alpha,K}(\rho) = \max_s\ (\rho - s)\, H_{\alpha,K}^{N-1}\big(\sigma_{\alpha,K}^{-1}(s)\big).$$

Proof of Proposition 2 (2/4): the envelope formula

Envelope theorem: only the direct effect of ρ counts, at $s = \sigma_{\alpha,K}(\rho)$:

$$\frac{\partial \pi_{\alpha,K}(\rho)}{\partial \rho} = H_{\alpha,K}^{N-1}(\rho).$$

Chain rule, with $H_{\alpha,K}(\rho_{\alpha,K}(\theta)) = F(\theta)$:

$$\frac{d\,\pi_{\alpha,K}(\rho_{\alpha,K}(\theta))}{d\theta} = F^{N-1}(\theta)\,\rho'_{\alpha,K}(\theta).$$

The lowest type never wins, $\pi_{\alpha,K}(\underline{\theta}) = 0$; integrate from $\underline{\theta}$:

$$\pi_{\alpha,K}(\theta) = \int_{\underline{\theta}}^{\theta} F^{N-1}(x)\,\rho'_{\alpha,K}(x)\,dx.$$

Profit depends on $\rho_{\alpha,K}$ only through its **slope**.

Proof of Proposition 2 (3/4): the first-order condition

With $C(q, \theta) = c(q)/\theta$:

$$\rho_{\alpha,K}(\theta) = \max_{Q} \left\{ \alpha Q - \frac{K\, c(Q)}{\theta} \right\}.$$

First-order condition; $Q^*_{\alpha,K}$ **rises in α, falls in K** (Lemma 2):

$$c'(Q^*_{\alpha,K}) = \frac{\alpha\, \theta}{K}.$$

Envelope theorem in θ, then substitute $K/\theta = \alpha/c'(Q^*_{\alpha,K})$:

$$\rho'_{\alpha,K}(\theta) = \frac{K}{\theta} \frac{c(Q^*_{\alpha,K})}{\theta} = \frac{\alpha}{\theta} \frac{c(Q^*_{\alpha,K})}{c'(Q^*_{\alpha,K})}.$$

Proof of Proposition 2 (4/4): compression

$\log c$ weakly concave: c'/c falls in q, so c/c' **rises** in q. In

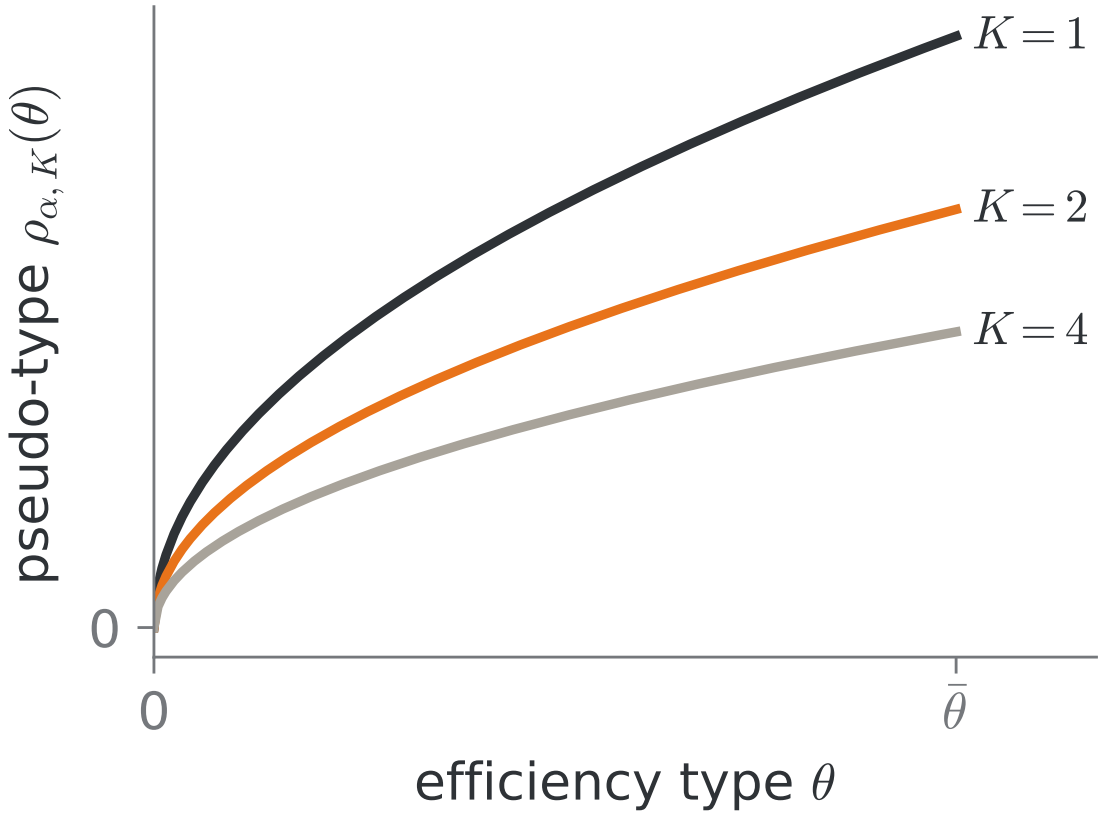
$$\rho'_{\alpha,K}(\theta) = \frac{\alpha}{\theta} \cdot \frac{c(Q^*_{\alpha,K})}{c'(Q^*_{\alpha,K})} :$$

More criteria ($K \uparrow$): α/θ is fixed and $Q^*_{\alpha,K}$ falls (Lemma 2), so ρ' falls: **compression**.

Insert into the envelope formula, $\pi_{\alpha,K}(\theta) = \int_{\underline{\theta}}^{\theta} F^{N-1}\rho'_{\alpha,K}$: rents fall too.

Corner $Q^*_{\alpha,K} = 0$ (when $\alpha\theta \leq K\, c'(0)$): $\rho = \rho' = 0$, weakly below any interior slope.

Power costs in a picture



Power costs $C = q^\gamma/(\gamma\theta)$, $\gamma = 3$: $\rho_{\alpha,K} \propto K^{-1/(\gamma-1)}$,
so each extra criterion flattens the pseudo-type: **compression**.

So far: fixed N, no entry cost

- More weight raises and an extra criterion lowers **every type's expected profit**: $\pi_{\alpha,K} \propto \alpha^2/K$ with quadratic costs.
- Preparing a bid is costly: forms, documentation, a technical project.

Conjecture: bidders see lower profits coming and some stop preparing bids.

Model, part 2: entry costs

The entry stage

Model

$M \geq 2$ identical potential suppliers. Each pays $f > 0$ to prepare a bid **before** learning θ; entrants then play the auction above, with reservation score 0.

Mixed entry equilibrium (Levin and Smith 1994, AER)

Firms are identical, so each enters with the same probability $p_{\alpha,K}$, set so that a firm is **indifferent**: the expected profit from entering, $\Pi_{\alpha,K}(p_{\alpha,K})$, equals the cost f.

Expected bidders

The number of entrants is $N_{\alpha,K} \sim \text{Binomial}(M, p_{\alpha,K})$, so $\mathbb{E}[N_{\alpha,K}] = M\, p_{\alpha,K}$.

Corners: $p_{\alpha,K} = 0$ or 1 when no such p exists. The equilibrium is the same whether or not the number of entrants is disclosed; Italian tenders do not disclose it.

The value of entering: definition

Definition

The **value of entering** $\Pi_{\alpha,K}(p)$ is an entrant's expected profit as a function of the entry probability p of the others: each of its $M-1$ rivals enters independently with probability p, and the entrant does not yet know its θ.

$$\Pi_{\alpha,K}(p) = \mathbb{E}_\theta\left[\pi_{\alpha,K}(\theta; p)\right].$$

- Profit gross of the entry cost f.
- Equilibrium entry: $\Pi_{\alpha,K}(p_{\alpha,K}) = f$.

The value of entering

Lemma 3 (each rival enters with probability p)

$$\Pi_{\alpha,K}(p) = \underbrace{(1-p)^{M-1}\rho_{\alpha,K}(\underline{\theta})}_{\text{sole entrant: level}} + \int_{\underline{\theta}}^{\bar{\theta}} \big(1-F(x)\big) \underbrace{\big(1-p+pF(x)\big)^{M-1}}_{\text{no rival outscores } x} \underbrace{\rho'_{\alpha,K}(x)}_{\text{slope}}\, dx.$$

A rival is harmless if it stays out *or* enters weaker.

The value of entering falls in p

Lemma 4 (more entry, less value)

$\Pi_{\alpha,K}(p)$ is decreasing in p.

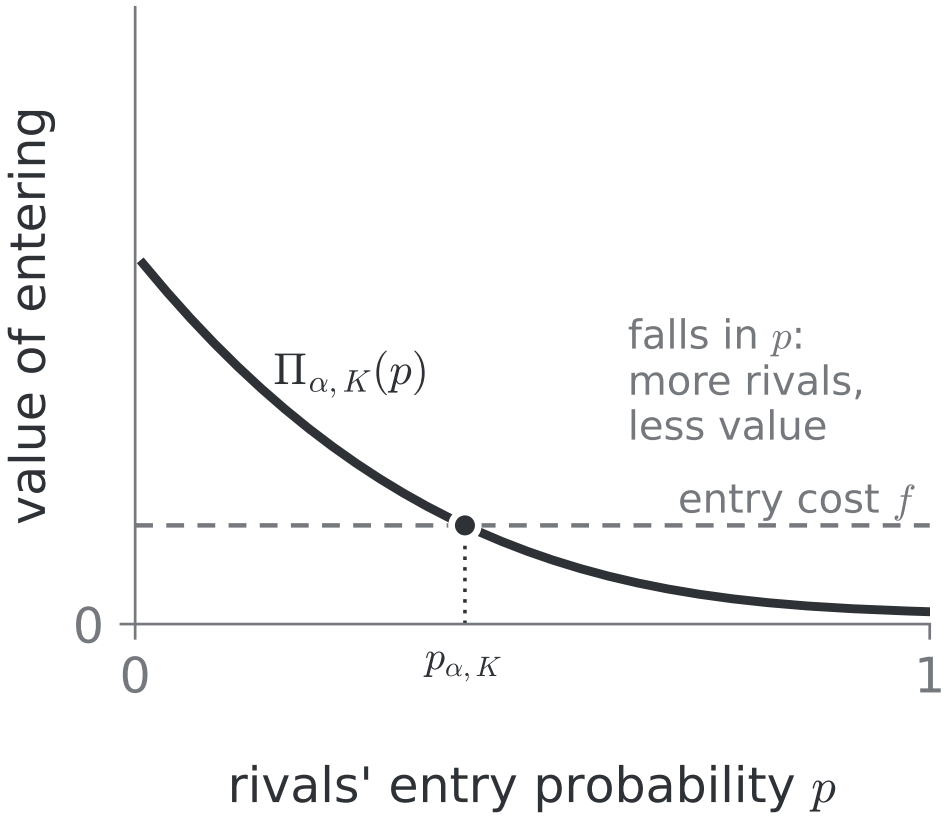
Recall Lemma 3:

$$\Pi_{\alpha,K}(p) = \underbrace{(1-p)^{M-1}}_{\text{falls in } p} \rho_{\alpha,K}(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} (1-F(x)) \underbrace{\left(1-p(1-F(x))\right)^{M-1}}_{\text{falls in } p} \rho'_{\alpha,K}(x)\,dx.$$

- Both weights fall in p; the level and the slope are ≥ 0.
- An extra entrant can only beat you.
- So at most one $p_{\alpha,K}$ solves $\Pi_{\alpha,K}(p_{\alpha,K}) = f$.

Uses: Lemma 3.

The value of entering in a picture



$\Pi_{\alpha,K}$ falls in p (Lemma 4), so it crosses f once: the equilibrium entry rate $p_{\alpha,K}$.

Illustration: quadratic costs, $\theta \sim U[1,2]$, $M = 4$.

The general result: more weight, more bidders

Proposition 3

For $\alpha' > \alpha$, the value of entering rises, $\Pi_{\alpha',K} \geq \Pi_{\alpha,K}$ at every p, and so does **entry**:

$$p_{\alpha',K} \geq p_{\alpha,K}, \qquad \mathbb{E}[N_{\alpha',K}] \geq \mathbb{E}[N_{\alpha,K}].$$

- No curvature condition: decompression in α holds for any single-crossing cost (Proposition 1), e.g. $C = c(q)/\theta$.
- Proof on the next slide.

Proof of Proposition 3

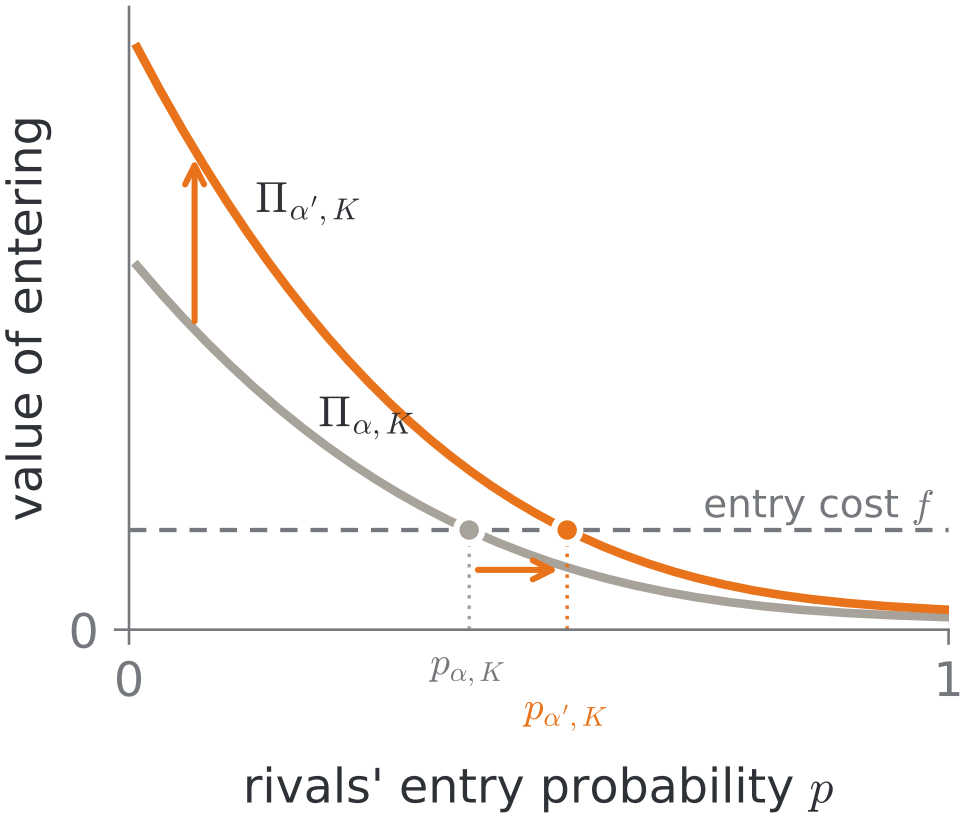
The value of entering (Lemma 3):

$$\Pi_{\alpha,K}(p) = \underbrace{(1-p)^{M-1}}_{\text{falls in } p} \quad \underbrace{\rho_{\alpha,K}(\underline{\theta})}_{\text{offset: rises in } \alpha} + \int_{\underline{\theta}}^{\bar{\theta}} (1-F(x)) \underbrace{\big(1-p+pF(x)\big)^{M-1}}_{\text{falls in } p} \quad \underbrace{\rho'_{\alpha,K}(x)}_{\text{slope: rises in } \alpha} \quad dx.$$

- Offset: $\partial \rho_{\alpha,K}(\underline{\theta})/\partial \alpha = Q^*_{\alpha,K} \geq 0$ (envelope theorem). Slope: Proposition 1.
- So $\Pi_{\alpha',K}(p_{\alpha,K}) \geq \Pi_{\alpha,K}(p_{\alpha,K}) = f$, and $\Pi_{\alpha',K}$ falls in p (Lemma 4): it reaches f at some $p_{\alpha',K} \geq p_{\alpha,K}$.
- $\mathbb{E}[N] = Mp$ rises too. □

Corner: if all enter at α, all enter at α'.

Proposition 3 in a picture



More weight lifts Π; it still falls in p, so it crosses f further right:

$$p_{\alpha',K} \geq p_{\alpha,K}.$$

Illustration: quadratic costs, $\theta \sim U[1,2]$, $M = 4$, $\alpha^2 = 3$, $K = 3$; $\alpha'^2 = 4.8$.

More criteria lower the level

Lemma 5 (more criteria lower every pseudo-type)

$\rho_{\alpha,K+1}(\theta) \leq \rho_{\alpha,K}(\theta)$ for every θ.

Derivation: since $C \geq 0$,

$$\rho_{\alpha,K+1}(\theta) = \max_Q\{\alpha Q - (K+1)C\} \leq \max_Q\{\alpha Q - KC\} = \rho_{\alpha,K}(\theta).$$

- Compression is about the **slope** of $\rho_{\alpha,K}$; this is about its **level**.
- The value of entering depends on both.

Uses only $C \geq 0$ (from $C(0,\theta) = 0$ and C increasing): no curvature condition, and nothing about F, N or equilibrium.

The value of entering falls in K

Lemma 6 (more criteria, less value)

If $(\alpha, K+1)$ compresses relative to (α, K), $\Pi_{\alpha,K+1} \leq \Pi_{\alpha,K}$ at every p.

Recall Lemma 3:

$$\Pi_{\alpha,K}(p) = (1-p)^{M-1} \underbrace{\rho_{\alpha,K}(\underline{\theta})}_{\text{level}} + \int_{\underline{\theta}}^{\bar{\theta}} \big(1-F(x)\big)\big(1-p+pF(x)\big)^{M-1} \underbrace{\rho'_{\alpha,K}(x)}_{\text{slope}}\, dx.$$

- The weights do not depend on K.
- Level: $\rho_{\alpha,K+1}(\underline{\theta}) \leq \rho_{\alpha,K}(\underline{\theta})$ by Lemma 5.
- Slope: $\rho'_{\alpha,K+1} \leq \rho'_{\alpha,K}$ by compression.

Uses: Lemmas 3 and 5, and compression.

The general result: more criteria, fewer bidders

Proposition 4

If $(\alpha, K+1)$ compresses relative to (α, K), **entry falls** with K:

$$p_{\alpha,K+1} \leq p_{\alpha,K}, \qquad \mathbb{E}[N_{\alpha,K+1}] \leq \mathbb{E}[N_{\alpha,K}].$$

- No functional form: any cost with compressing pseudo-types. The multiplicative case follows as a corollary.
- Proof on the next slide.

Proof of Proposition 4

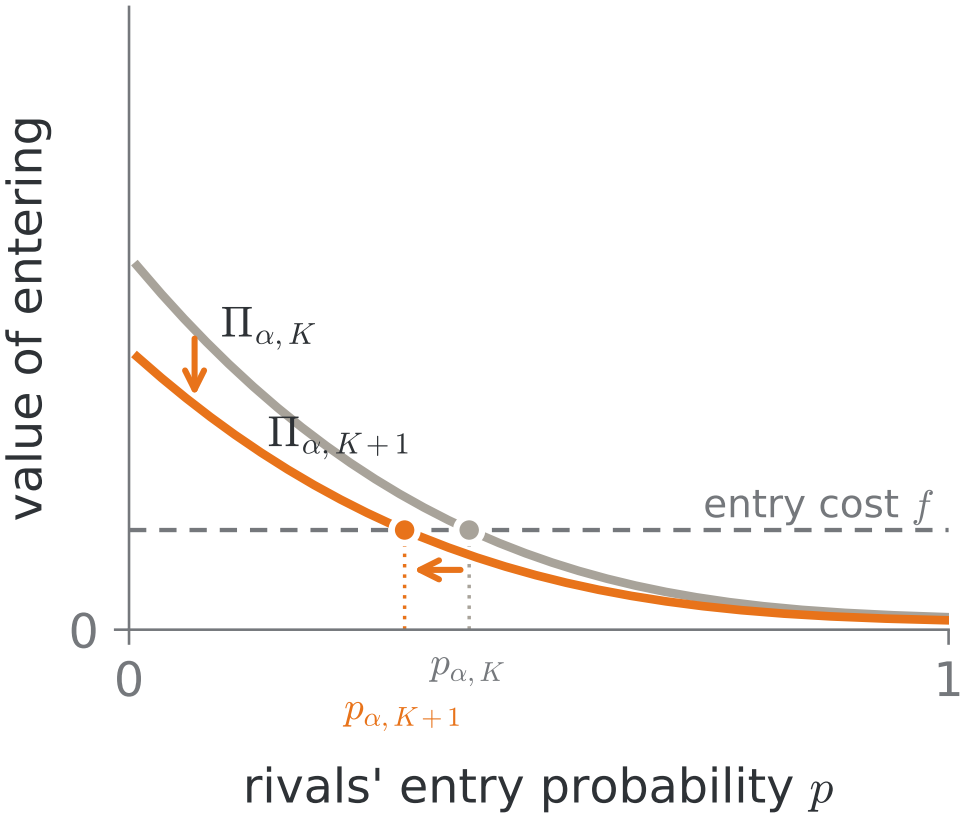
The same formula (Lemma 3), now in K:

$$\Pi_{\alpha,K}(p) = \underbrace{(1-p)^{M-1}}_{\text{falls in } p} \quad \underbrace{\rho_{\alpha,K}(\underline{\theta})}_{\text{offset: falls in } K} + \int_{\underline{\theta}}^{\bar{\theta}} \big(1-F(x)\big) \underbrace{\big(1-p+pF(x)\big)^{M-1}}_{\text{falls in } p} \underbrace{\rho'_{\alpha,K}(x)}_{\text{slope: falls in } K} dx.$$

- Offset: Lemma 5. Slope: compression. So $\Pi_{\alpha,K+1} \leq \Pi_{\alpha,K}$ (Lemma 6).
- So $\Pi_{\alpha,K+1}(p_{\alpha,K}) \leq f$, and $\Pi_{\alpha,K+1}$ falls in p (Lemma 4): it reaches f at some $p_{\alpha,K+1} \leq p_{\alpha,K}$.
- $\mathbb{E}[N] = Mp$ falls too. □

Corner: if nobody enters at K, nobody enters at $K+1$.

Proposition 4 in a picture



One more criterion lowers Π; it still falls in p, so it crosses f further left:

$$p_{\alpha,K+1} \leq p_{\alpha,K}.$$

Illustration: quadratic costs, $\theta \sim U[1,2]$, $M = 4$, $\alpha^2 = 3$, $K = 3 \to 4$.

The multiplicative model

Corollary

If $C(q,\theta) = c(q)/\theta$ with c increasing, strictly convex and $\log c$ **weakly concave**, then **entry falls with** K but **rises with** α:

$$p_{\alpha,K+1} \leq p_{\alpha,K} \leq p_{\alpha',K} \qquad \text{for } \alpha' > \alpha.$$

Proof.

- K: Proposition 2 gives compression, so Proposition 4 applies.
- α: Proposition 3, since $C_{q\theta} = -c'(q)/\theta^2 \leq 0$.

Covers all power costs $c(q) = q^\gamma$, $\gamma > 1$.

Quadratic costs: entry depends on α^2/K

Result

With $C(q,\theta) = q^2/(2\theta)$, $\rho_{\alpha,K}(\theta) = \frac{\alpha^2}{2K}\,\theta$, so

$$\Pi_{\alpha,K}(p) = \frac{\alpha^2}{2K}\,B_M(p),$$

$$B_M(p) = (1-p)^{M-1}\underline{\theta} + \int_{\underline{\theta}}^{\bar{\theta}} \big(1-F\big)\big(1-p+pF\big)^{M-1} dx.$$

Entry solves $B_M(p_{\alpha,K}) = 2Kf/\alpha^2$: it depends on (α, K) only through α^2/K.

- B_M depends only on F, M and p, and falls in p (Lemma 4).
- Doubling $K =$ cutting α by the factor $1/\sqrt{2}$.

Holding entry fixed: points per extra criterion

Criteria	Technical points (of 100) that keep entry unchanged	
$K \to K+1$	one step, from 70	cumulative, from 70 at $K = 2$
$2 \to 3$	74.1	74.1
$3 \to 4$	72.9	76.7
$4 \to 5$	72.3	78.7
$5 \to 6$	71.9	80.2

- One more criterion costs 2–4 points; less when there are already many.
- The steps add up: going from 2 to 5 criteria needs about **79 points**.

Quadratic costs. α = technical points / price points (70/30 gives $\alpha = 2.33$). Entry fixed means α^2/K fixed: $\alpha' = \alpha\sqrt{(K+1)/K}$.

Takeaways (1/2): fact and channel

- **Fact.** Italian green criteria add criteria, not weight: within buyer, **about 4.5 fewer bidders**. Conditional associations, not causal estimates.
- **A puzzle: why does adding green reduce entry?**
- **Channel: pseudo-type compression**. A new criterion is a new cost dimension $\Rightarrow$ less effort per dimension $\Rightarrow$ efficiency gaps matter less $\Rightarrow$ **lower rents** $\Rightarrow$ fewer firms pay to bid.
- More quality weight does the opposite: **pseudo-type decompression** raises entry.

Takeaways (2/2): quadratic costs

- Four ratios:
  - quality Q^* depends on α/K;
  - pseudo-type ρ depends on α^2/K;
  - profit π depends on α^2/K;
  - bidders $\mathbb{E}[N]$ depend on α^2/K.
- Mnemonic: holding α^2/K fixed holds entry.
  - Doubling K needs $\alpha \times \sqrt{2}$, i.e. $+41\%$.
  - $K = 2 \to 5$ at 70/30: raise quality to 79 points.

Adding criteria is not free for competition.

Green don’t fit?

Then you must quit!