Scoring and Favoritism in Optimal Procurement Design

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Scoring auctions

- *Scoring auctions* are widely used for procurement in many countries
- In a scoring auction, potential sellers submit multidimensional price-quality bids and then the buyer selects a winner using some predetermined scoring rule (a formula that converts a multidimensional bid into a single number)
- Example: "A+B" auctions for road construction in California, where "A" is price and "B" is speed of construction
- A popular choice of the scoring rule is a *quasilinear score*

$$S(q, p) = s(q) - p$$

(q is quality, p is price)

Che (1993)

- In an influential paper (Che, 1993), Che proves that in a "standard" setting (one-dimensional types, symmetry, well-behaved cost and utility functions) a scoring auction with a carefully chosen quasilinear scoring rule is a buyer-optimal mechanism for procurement
- The optimal quality score function $s^*(q)$ is not equal to true buyer's utility $v(q)$; $s^*(q)$ is flatter than $v(q)$
  - The buyer wants to depress firms' incentives to increase quality relative to the efficient level in order to save on information rents
- So far much of theoretical literature on scoring auctions largely follows Che's framework (e.g., Branco (1997), Asker and Cantillon (2008), Asker and Cantillon (2010), Nishimura (2015))

Our focus: Investment costs

- Importantly, in the current literature following Che (1993), it is assumed that *only the winner of the contract* incurs costs $c(q, \theta)$
  - In other words, all costs are *production costs*
- In contrast, in reality a substantial share of costs has to be incurred before the scoring auction, i.e. *by every participant*
  - Developing and improving the product
  - Gaining experience
  - Gathering information
- I.e., real-world scoring auctions inevitably have an all-pay component – *investment costs* that have to be incurred regardless of winning
- Also, these investments are *independent*, in that a firm's investment may not depend on other firms' private information
- Q: *How would an optimal mechanism for procurement change in such an environment?*

Plan for today:

1. I will show a full characterization of optimal symmetric mechanisms when there are both production and investment costs including the optimal scoring formula
2. I will show a number of results saying that in general the optimal mechanism in our setting is asymmetric (non-anonymous) even with ex-ante symmetric bidders
3. I will give a partial characterization of optimal (potentially asymmetric) mechanisms for $n = 2$. An optimal mechanism combines scoring with favoritism in a non-trivial way

Relation to literature

1. **Procurement with Endogenous Quality (via Scoring Auctions).** Che (1993), Asker and Cantillon (2008), Asker and Cantillon (2010), Lewis and Bajari (2011), Decarolis et al. (2016), ...
2. **Procurement with Exogenous Quality.** Manelli and Vincent (1995), Lopomo et al. (2022), ...
3. **Auctions with Investment Before Learning Type.** Tan (1992), Piccione and Tan (1996), Arozamena and Cantillon (2004), ...
4. (Closest.) **Auctions with Investment After Learning Type.** Celik et al. (2009), Zhang (2017), Gershkov et al. (2021), ...
5. **(Enormous) literature on contests and tournaments** in which all costs are all-pay costs, i.e. Drugov and Ryvkin (2017)

In our paper, we blend literatures 1. and 4.

Model

- A single buyer (principal) wishes to procure a unit of some good/service, for which there are n potential suppliers (firms)
- $q_i \in \mathbb{R}$, the quality of firm's i good, is endogenous and contractible
- $\theta_i \in [0, 1]$ is a cost (dis)-efficiency parameter of firm i, a higher θ_i corresponds to higher marginal costs of quality
- θ_i are iid from distribution F with density f
- The costs consist of two components:
  - Production costs $c^P(q, \theta)$. *Only the winner incurs these costs*
  - Investment costs $c^I(q, \theta)$. *Every participant incurs investment costs* New for this subliterature
- Cost functions satisfy several regularity conditions...
- Buyer's utility is $v(q)$ before transfers

Outcomes and mechanisms

- z_i is the probability that firm i gets the contract
- The good must be procured in any case, i.e. $\sum_i z_i = 1$
- t_i is the (expected) transfer to firm i
- Because the qualities are observable and contractible, we would like to include qualities directly as determinants of allocation and transfers
- Thus, our definition of a mechanism is somewhat non-standard
- A mechanism $\mathcal{M}$ is an n-tuple of *message sets* $(M_1, M_2, \ldots, M_n)$ and functions
  - $z_i(\mathrm{m}, \mathrm{q})$, $i = 1, \ldots, n$ — allocation;
  - $t_i(\mathrm{m}, \mathrm{q})$, $i = 1, \ldots, n$ — transfers,

  where m is the vector of all messages and q is the vector of all qualities

Timing

1. The buyer announces a mechanism $\mathcal{M}$
2. Each firm i privately learns its type θ_i
3. Each firm i chooses its quality q_i
4. Each firm i submits a message $m_i \in M_i$
5. The buyer observes all q and m and implements allocation and transfers according to $\mathcal{M}$
6. Payoffs realize

- Important: given this timing, the equilibrium quality q_i may depend only on own type θ_i while allocation z_i and transfers t_i may depend on the types of all firms through the messages sent. That is, the investment decisions are independent
- That is, we write $q_i(\theta_i)$ and $z_i(\theta)$, $t_i(\theta)$ for equilibrium outcomes
- This makes the problem non-standard and mathematically non-trivial

Outcome functions and Revelation Principle

- The problem is to find a mechanism $\mathcal{M}^*$ that maximizes the buyer's utility
- We assume that the firms will play a Bayes-Nash Equilibrium (BNE) of $\mathcal{M}$
- Every $\mathcal{M}$ and a BNE $(m_i^*(\theta_i), q_i^*(\theta_i))$, $i = 1, \dots, n$ of $\mathcal{M}$ induce *outcome functions* $z_i(\theta), t_i(\theta), q_i(\theta_i)$ that satisfy Bayesian IC constrains
- Because the objective depends on $\mathcal{M}$ only through the outcome functions $z_i(\theta), t_i(\theta), q_i(\theta_i)$, we can optimize directly over them (Revelation Principle)
- So our plan of attack is:
  1. Find optimal outcome functions $z_i^*(\theta), t_i^*(\theta), q_i^*(\theta_i)$
  2. Pinpoint a (non-direct) mechanism $\mathcal{M}^*$ that implements $z_i^*(\theta), t_i^*(\theta), q_i^*(\theta_i)$

Problem formulation

- The buyer's utility u_b is $u_b(z,t,q) = \sum_{i=1}^{n} \left(v(q_i)z_i - t_i\right)$
- A seller's utility is $u_{is}(z,t,q_i,\theta_i) = t_i - c^P(q_i,\theta_i)z_i - c^I(q_i,\theta_i)$
- The optimization problem over outcome functions can be written as follows:

$$(\text{P1}) \quad U = \max_{z,t,q} \mathbb{E}\left(u_b(z(\theta),t(\theta),q(\theta))\right),$$

subject to $z_i(\theta) \geqslant 0$, $\sum_{i=1}^{n} z_i(\theta) = 1$ and the standard Bayesian IC and IR constraints

$$\theta_i \in \arg\max_{\theta'} \mathbb{E}_{\theta_{-i}}\left(u_{is}(z(\theta',\theta_{-i}),t(\theta',\theta_{-i}),q_i(\theta'),\theta_i)\right),$$

$$\mathbb{E}_{\theta_{-i}}\left(u_{is}(z(\theta_i,\theta_{-i}),t(\theta_i,\theta_{-i}),q_i(\theta_i),\theta_i)\right) \geqslant 0,$$

for all θ_i in the support.

Problem reformulation

By the standard envelope argument, we can get rid of transfers t: they are pinned down by z and q ("revenue equivalence"). Using this, we proceed to the relaxed problem

$$\text{(P2)} \quad U = \max_{z(\cdot),q(\cdot)} \mathbb{E} \sum_{i=1}^{n} \Big(v(q_i) z_i - \tilde{c}^P(q_i, \theta_i) z_i - \tilde{c}^I(q_i, \theta_i) \Big),$$

$$\text{s.t.} \quad z_i(\theta) \geqslant 0, \quad \sum_{i=1}^{n} z_i(\theta) = 1,$$

$$\text{where } \tilde{c}^P = c^P + \frac{\partial c^P}{\partial \theta_i} \frac{F(\theta_i)}{f(\theta_i)}, \quad \tilde{c}^I = c^I + \frac{\partial c^I}{\partial \theta_i} \frac{F(\theta_i)}{f(\theta_i)},$$

ignoring non-local IC constraints

$\tilde{c}^P$ and $\tilde{c}^I$ are *virtual* production and investment costs

Optimal symmetric mechanisms

$$U_b = \mathbb{E} \sum_{i=1}^{n} \left(\left(v(q_i) - \tilde{c}^P(q_i, \theta_i) \right) z_i - \tilde{c}^I(q_i, \theta_i) \right)$$

- We first optimize over z_i: the buyer will award the contract to a firm for which the virtual production surplus

  $$x(q(\theta_i), \theta_i) := v(q_i(\theta_i)) - \tilde{c}^P(q_i(\theta_i), \theta_i)$$

  is maximal
- Under appropriate regularity conditions, this will be the firm with the lowest θ_i
- Without the investment (all-pay) costs, as in Che (1993), the optimal symmetric quality schedule $q(\theta)$ would be the one maximizing $x(q, \theta)$ pointwise. With investment costs, it will be different

Optimal symmetric mechanisms

- Denoting by $\theta_{(1)}$ the lowest type, the buyer's payoff is

$$U_b = \mathbb{E}\left(x(q(\theta_{(1)}), \theta_{(1)}) - \sum_{i=1}^{n} \tilde{c}^I(q_i, \theta_i)\right)$$

- We cannot optimize this integral pointwise right away; but for symmetric mechanisms we can still do this after a transformation
- Given $F(\cdot)$, the pdf of $\theta_{(1)}$ is given by $n(1 - F(\theta))^{n-1} f(\theta)$.
- By symmetry, the buyer's payoff becomes

$$U = n \int \left(x(q(\theta), \theta))(1 - F(\theta))^{n-1} - \tilde{c}^I(q(\theta), \theta)\right) f(\theta) d\theta. \tag{1}$$

Optimal symmetric mechanisms

Proposition 1

The quality schedule $q^*(\theta)$ solving the relaxed problem (P2) under the symmetry constraint $q_i(\theta) \equiv q_j(\theta)$ for all i, j is determined by maximizing (1) pointwise. That is, $q^*(\theta)$ maximizes

$$(v(q) - \tilde{c}^P(q,\theta))(1 - F(\theta))^{n-1} - \tilde{c}^I(q,\theta)$$

over q for each θ. Moreover, $q^*(\theta)$ is decreasing.

Optimal symmetric mechanisms: implementation

- Under which mechanism $\mathcal{M}^*$ do we achieve the optimal quality schedule $q^*(\theta)$ in equilibrium?
- We show that one may take as $\mathcal{M}^*$ a first-score scoring auction with a carefully chosen scoring function $S(q,p) = s(q) - p$
- It is easy to see that by the every firm's first-order condition, the scoring function $s(q)$ must satisfy

$$s'(q) \equiv C_q^P(q, \theta(q)) + \frac{C_q^I(q, \theta(q))}{(1 - F(\theta(q)))^{n-1}}, \tag{2}$$

  where $\theta(q)$ is the inverse of $q(\theta)$
- But it's far from immediate that $q(\theta)$ will be globally optimal for each firm in such an auction
- Recall that in a scoring auction a strategy is 2-dimensional (q, p) and we have to check all joint quality-price deviations....

Optimal symmetric mechanisms: implementation

Surprisingly, we find that no additional fancy conditions (on Hessian, etc) are needed for all those deviations to be unprofitable!

Proposition 2

For any decreasing quality schedule $q(\theta)$ with inverse $\theta(q)$, including the quality schedule $q^*(\theta)$ identified in Proposition 1, the first-score auction with the score $S(q,p) = s(q) - p$, where $s(q)$ satisfies (2), has a BNE in which the quality strategy of every firm is $q(\theta)$.

Optimal symmetric mechanisms: comparative statics

Proposition 3

Let the investment costs be parametrized as $c^I = \beta h(q, \theta)$. The optimal symmetric quality schedule $q^*(\theta, \beta, n)$ is:

1. *Decreasing in the size of the investment costs β;*
2. *Decreasing in the number of bidders n.*

The dependence on n is in contrast to Che (1993) where the optimal quality does not depend on n

Optimal symmetric mechanism: comparative statics of optimal score $s^*(q)$

- This question is subtler, and the answer actually depends on whether production or investment costs are subject to more informational asymmetry

Proposition 4

Suppose $c^P(q,\theta) = \theta^{E_1} g_1(q)$ and $c^I(q,\theta) = \beta \cdot \theta^{E_2} g_2(q)$ where $E_1, E_2 > 0$ and $g_1(q)$, $g_2(q)$ are some well-behaved functions. Suppose $F(\theta) = \theta^{\frac{1}{d}}$ for some $d > 0$. Denote by $s_q^*(q)$ the slope of optimal score. Then:

1. *If $E_1 > E_2$ then $s_q^*(q)$ increases in β and n at every q;*
2. *If $E_1 < E_2$ then $s_q^*(q)$ decreases in β and n at every q.*

Asymmetric mechanisms can perform better!

- We parametrize costs as $C^P(q, \alpha\theta)$ and $C^I(q, \alpha\theta)$ where α is the degree of importance of private information

Proposition 5

There exists $\overline{\alpha} > 0$ such that for all $\alpha \in [0, \overline{\alpha}]$ the optimal symmetric mechanism is not an optimal mechanism.

- Proved by considering a mechanism when we contract only with one firm (single-sourcing)
- More generally, the main trade-off here is between the ex-post efficiency and the avoidance of investment costs duplication

Asymmetric mechanisms can perform better!

- Now we provide a result that holds for all values of α, not only small ones
- Denote by γ the elasticity of investment costs with respect to quality q at $q = 0$

Proposition 6

Suppose $\gamma > n$ (recall that n is the number of bidders). Then the optimal symmetric mechanism is not an optimal mechanism.

- Proved by considering a mechanism every bidder but one (the favored) is excluded when her θ is higher than a certain threshold
- Intuition: we want an asymmetric mechanism when the problem of duplication of investment costs is severe (γ is high) or when the best theta is bad enough anyway so it's not important to reveal it (n is low)

Asymmetric mechanisms can perform better even in the limit!

Proposition 7

There exists $\overline{\alpha} > 0$ such that for all $\alpha \in [0, \overline{\alpha})$, the sequence of optimal symmetric mechanisms M_n^{symm} is not asymptotically optimal.

- Proved by considering a mechanism when we contract only with one firm (single-sourcing)

Optimal mechanisms without restriction to symmetric ones

- If the optimal symmetric mechanism is not optimal, which is?
- In general, the analysis is hard. The mechanism design problem is here mathematically nontrivial as:
  - One cannot use pointwise integral maximization (as is common in mechanism design) due to the independent investment decisions
  - The objective is not concave
- We restrict attention to situations with:
  - two bidders ($n = 2$)
  - $C^P(q, \theta) = \alpha\theta$
  - $C^I(q, \theta) = g(q)$ with $g'(q) > 0$, $g''(q) > 0$
  - $v(q) = q$ (WLOG with one-dimensional quality)
- We shall show that in this setting the form of optimal mechanism depends on the degree of convexity of marginal investment costs
- In this setting, the techniques of Zhang (2017) are applicable
- We developed a more general technique but describing it requires a separate paper..

Optimal mechanisms without restriction to symmetric ones

- Denote by $J(\theta) = \theta + \frac{F(\theta)}{f(\theta)}$ the standard virtual type. (J is increasing under our assumptions.)
- Define $\xi_F(z) := 1 - J(F^{-1}(1-z))$ (will be also increasing)

Definition 1

We say that marginal investment costs are sufficiently convex iff the function $q \to \alpha\xi_F(g'(q)) - q$ is strictly quasi-convex. We say that marginal investment costs are sufficiently concave iff the function $q \to \alpha\xi_F(g'(q)) - q$ is strictly quasi-concave.

- If θ_i are uniformly distributed ($F(\theta) = \theta$):
  - MC are sufficiently convex just iff MC are convex ($g'''(q) > 0$)
  - MC are sufficiently concave just iff MC are concave ($g'''(q) < 0$)

Optimal mechanisms without restriction to symmetric ones: main result

Theorem 1

Suppose $n = 2$, $C^P(q,\theta) = \alpha\theta$, $C^I(q,\theta) = g(q)$ and $v(q) = q$. Then:

1. *If the marginal costs are sufficiently convex, there exists a $\theta_0 \in [0,1]$ such that an optimal pair of quality schedules is*

$$q_1^*(\theta) = \begin{cases} q_{symm}(\theta), & \theta < \theta_0; \\ q_{symm}(\theta_0), & \theta > \theta_0; \end{cases} \quad q_2^*(\theta) = \begin{cases} q_{symm}(\theta), & \theta < \theta_0; \\ 0, & \theta > \theta_0. \end{cases} \tag{3}$$

2. *If the marginal costs are sufficiently concave, there exists a $\theta_0 \in [0,1]$ such that an optimal pair of quality schedules is*

$$q_1^*(\theta) = \begin{cases} q_{symm}(0), & \theta < \theta_0; \\ q_{symm}(\theta), & \theta > \theta_0; \end{cases} \quad q_2^*(\theta) = \begin{cases} q_{symm}(\theta_0), & \theta < \theta_0; \\ q_{symm}(\theta), & \theta > \theta_0. \end{cases} \tag{4}$$

In both cases, bidder 1 is the "favored" bidder while bidder 2 is the "unfavored" one.

Optimal mechanism with sufficiently convex marginal costs

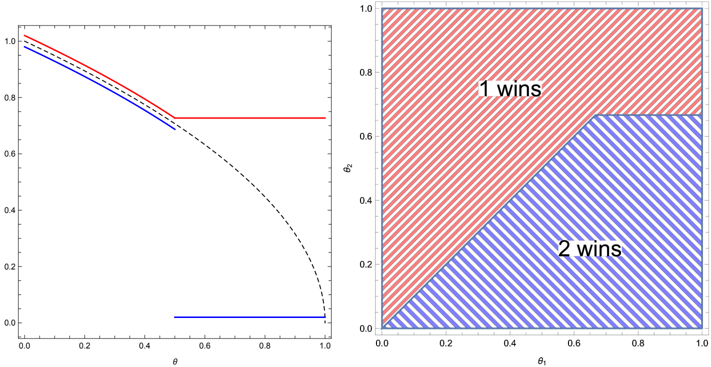


Figure: Optimal quality for the (favored) firm 1 and (unfavored) firm 2 (left); allocation (right) with sufficiently convex marginal costs

Optimal mechanism with sufficiently concave marginal costs

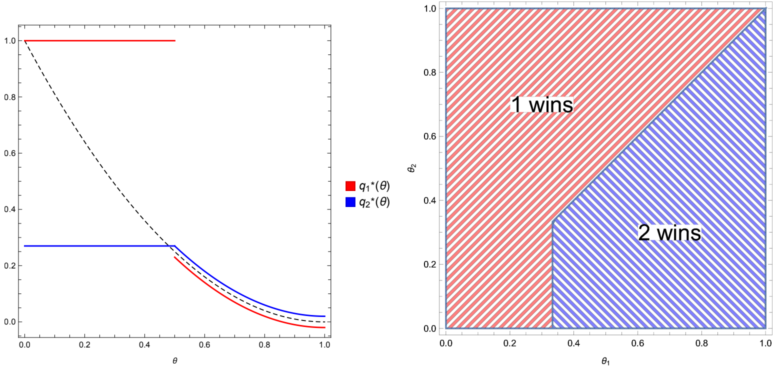


Figure: Optimal quality for the (favored) firm 1 and (unfavored) firm 2 (left); optimal allocation with sufficiently concave marginal costs (right)

Implementation: score floors

Proposition 8

The optimal quality schedules given by (3) *can be implemented in a modified first-score scoring auction in which:*

- *the quasi-linear score* $S(q, p) = s(q) - p$ *is used;*
- *the favored bidder faces a reserve score (score floor) of* S_1^r *That is, her score* S_1 *counts only if* $S_1 \geqslant S_1^r$.
- *the unfavored bidder faces a reserve score (score floor) of* $S_2^r > S_1^r$. *That is, her score* S_2 *counts only if* $S_2 \geqslant S_2^r$.
- *the favored bidder gets a bonus* $B_1 > 0$ *if her score exceeds the unfavored bidder's reserve score* S_2^r, *regardless of whether the favored bidder wins or not.*

Implementation: score ceilings

Proposition 9

The optimal quality schedules given by (4) *can be implemented in a modified first-score scoring auction in which:*

- *the quasi-linear score* $S(q, p) = s(q) - p$ *is used;*
- *the unfavored bidder faces a score ceiling of* $\tilde{S}$*. That is, the unfavored bidder's score* S_2 *does not count if* $S_2 > \tilde{S}$*.*
- *the favored bidder faces any score ceiling weakly higher than* $\tilde{S}$*. In particular, the favored bidder may face no score ceiling.*
- *ties are resolved in favor of the favored bidder;*
- *Upon winning with score* $\tilde{S}$*, the favored bidder must pay the buyer a kickback of* $T > 0$

The roles of bonus and kickback

- The kickback here is no way due to corruption which is absent from our model as the buyer and the auctioneer are one and the same.
- The role of the side-payments (bonus and kickback) is to provide correct incentives to "middle" types of the favored bidder.
  - Without the bonus, θ_0 and some more efficient types would opt out of the "fierce competition" high-scores, choosing to win with score S_1^r and with comfortable probability of $1 - F(\theta_0)$ instead. Thus, the bonus is needed to achieve the efficient symmetric competition among types $\theta < \theta_0$.
  - The kickback is needed to ensure that the types $\theta_1 > \theta_0$ of the favored bidder do not rush to win for sure with the score $\tilde{S}$ and instead maintain efficient symmetric competition with the types $\theta_2 > \theta_0$ bidder 2 in the low-score range.

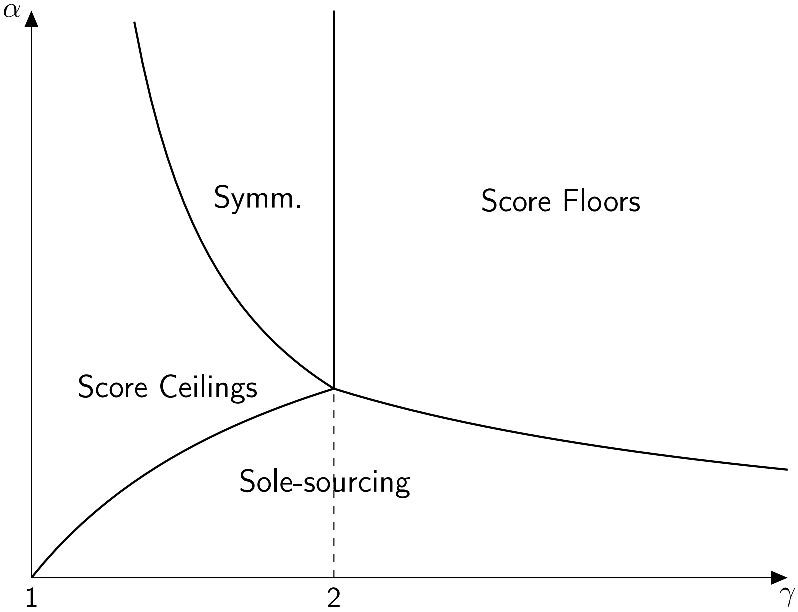
Example: optimal mechanism with constant-elasticity costs

Proposition 10

Suppose $n = 2$, $F(\theta) = \theta$, $C^P = \alpha\theta/2$, $C^I = q^\gamma/\gamma$ where $\alpha \geqslant 0$, $\gamma > 1$. Then, an optimal mechanism is:

- *sole-sourcing if $\alpha < \min\left\{2 - \frac{2}{\gamma}, \frac{2}{\gamma}\right\}$;*
- *a scoring auction with discriminatory score ceilings if $\gamma \leqslant 2$ and $2 - \frac{2}{\gamma} < \alpha < \frac{1}{\gamma - 1}$;*
- *a non-discriminatory scoring auction (the optimal symmetric mechanism) if $\gamma < 2$ and $\frac{1}{\gamma-1} < \alpha$;*
- *a scoring auction with discriminatory score floors (reserve scores) if $\gamma > 2$ and $\frac{2}{\gamma} < \alpha$.*

Example: optimal mechanism with constant-elasticity costs



Comparative statics

Regardless of the form of optimal mechanism, it becomes *more symmetric* when α grows. Intuition: ex-post efficiency is more important when private information varies more wildly. So:

$$\text{Information asymmetry} \uparrow \implies \text{an optimal mechanism's symmetry} \uparrow$$

Proposition 11

Suppose $n = 2$, $C^P = \alpha\theta$, $C^I = g(q)$ *and* θ_i *is not necessarily uniformly distributed. Then, for the optimal score floors mechanism (including boundary cases), the optimal threshold* θ_0^* *is weakly increasing in the importance of private information* α*. Thus, the optimal score floors mechanism becomes more symmetric as* α *grows.*

Proposition 12

Suppose $n = 2$, $C^P = \alpha\theta$, $C^I = g(q)$ *and* θ_i *is not necessarily uniformly distributed. Then, for the optimal score ceilings mechanism (including boundary cases), the optimal threshold* θ_0^* *is weakly decreasing in the importance of private information* α*. Thus, the optimal score ceilings mechanism becomes more symmetric as* α *grows.*

Optimal vs. (Constrained)-Efficient Mechanisms

Proposition 13

Suppose $n = 2$, $C^P = \alpha\theta$, $C^I = g(q)$ with either $g'''(q) > 0$ for all q or $g'''(q) < 0$ for all q, and $F(\theta) = \theta$. Then, the efficient mechanism is weakly more asymmetric (exhibits weakly more favoritism) than the buyer-optimal mechanism.

- Intuition: A social planner acts as if she faces a lower α as she consider only costs c themselves but not the virtual costs $\tilde{c} = c + c_\theta F/f$ and costs are less responsive to private information than virtual costs

Allow-k-bidders mechanisms

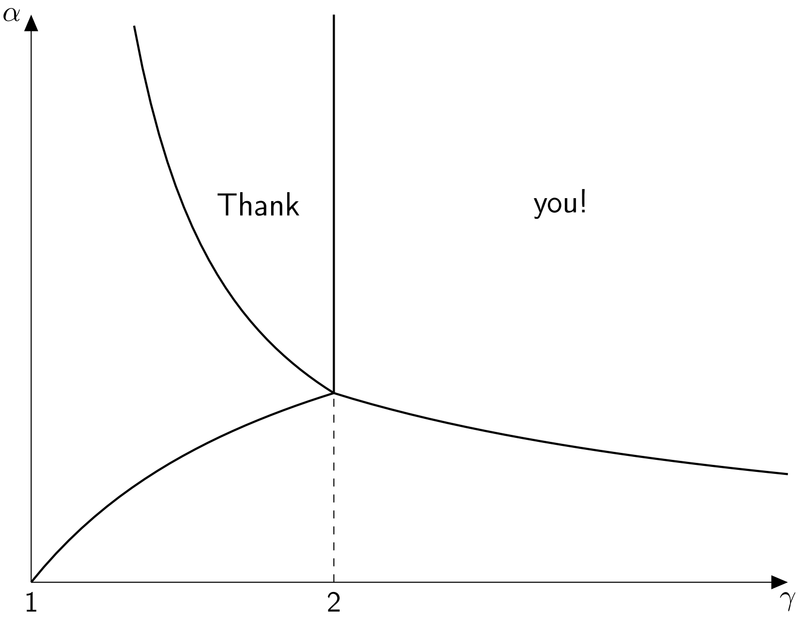
- One interesting family of asymmetric mechanisms is mechanisms where the principal allows only $k \leqslant n$ bidders to enter, and employs the optimal symmetric mechanism for these k bidders.
- Such mechanisms may be more practical than arbitrary asymmetric mechanisms since this particular kind of asymmetry may be less salient, and thus on the surface such mechanisms may look more "fair"
- How many bidders should we allow to enter?

Proposition 14 (One-or-all)

Suppose $C^P = \alpha\theta$, $C^I = g(q)$ where $g(0) = 0$, $g'(q)$ is strictly increasing and $\frac{g(q)}{\sqrt{g'(q)}}$ is also strictly increasing. Suppose also that $F(\theta) = \theta$. Then, it is optimal for the principal to allow either one or all bidder to enter, that is, $k^* \in \{1, n\}$.

Concluding remarks

- We set-up and partially solved a mechanism design problem of procurement with endogenous and contractible quality in which a part of costs if *all-pay*
- When restricting attention to symmetric mechanisms, we showed that a scoring auction with a carefully chosen (novel) scoring rule is optimal
- We showed that in general an optimal mechanism for symmetric players is asymmetric
- In a restricted setting, we characterized optimal quality schedules and offered a (novel) implementation of those via a modified first-score auctions
- Further work: full characterization of optimum for $n > 2$ (hard)
- Ideally we would like to estimate empirically the gain from an asymmetric mechanism relative to the optimal symmetric one and the gain from the correct scoring functions relative to the status-quo scoring function



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you!
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