Optimal Robust Double Auctions

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Double auctions

The paper is about to be split into two:

- Myerson-optimal robust* double auctions
- Price-path-optimal robust* double auctions

The first part is the maximization of (scalar) expected revenue subject to robust mechanisms, traditionally done with one-dimensional types.

The second part is the maximization of something else (is there even stuff to optimize?) given the already Myerson-optimal mechanism, which will be done with full-support types.

* means ex-post constraints

Part 1: Myerson-optimal robust direct mechanism

Consider a classical single-crossing (but non-linear) consumption utility $u(\theta, q)$ and quasi-linear payoff $u(\theta, q) - t(\theta)$ structure. Let the type θ and allocation q be 1-dim. A leading example - is electricity.

Leading example: $u(\theta, q) = \theta q - \mu q^2$, where μ is known

What is an optimal (revenue-maximizing) robust mechanism?

- denote (equilibrium) surplus as $s(\theta) = u(\theta, q(\theta)) - t(q(\theta), \theta_{-i})$
- maximize average $u(\theta, q) - s(\theta)$
- ex post IC constraint $s(\theta) = \max_q [u(\theta, q) - t(q, \theta_{-i})]$
- ex post IR constraint $s(\theta) \geqslant u(\theta_i, 0)$

Standard virtualization techniques apply

$$v(\theta_i, q|\theta_{-i}) = (\theta - \frac{I(\theta_i > wot(\theta_{-i})) - F(\theta_i)}{f(\theta_i)})q - \mu q^2$$

where wot(θ_{-i}) is the worst-off type.

Problems:

- is this even a "private utility"?
- virtual utility is (downwards) discontinuous in own type
- worst-off type is endogenous to the sought mechanism
- worst-off type is conditional on types of others

There will be ironing. Seems like one agent will do the ironing while other agents do the ironing, all riding on top of a fixed point...

Should we panic already?

The same thing that caused trouble (the ex-post) will be our salvation.

Lemma 1: let tet(θ_{-i}) be the type excluded from trade, that is, who trades exactly zero, then it is one of the worst-off types wot(θ_{-i}).

$$\text{in other words, } tet(\theta_{-i}) \subset wot(\theta_{-i}).$$

Note that this trick only works for the ex-post constraints. For example, the worst-off interim type does not have to trade zero even in expectation unless there is extreme symmetry in the model.

Thus virtual utility

$$v(\theta_i, q|\theta_{-i}) = (\theta - \frac{I(\theta_i > wot(\theta_{-i})) - F(\theta_i)}{f(\theta_i)})q - \mu q^2$$

can be rewritten wlog

$$v(\theta_i, q|\theta_{-i}) = (\theta - \frac{I(\theta_i > tet(\theta_{-i})) - F(\theta_i)}{f(\theta_i)})q - \mu q^2$$

or, put differently

$$v(\theta_i, q|\theta_{-i}) = (\theta - \frac{I(q > 0) - F(\theta_i)}{f(\theta_i)})q - \mu q^2$$

So this is our virtual utility

$$v(\theta_i, q|\theta_{-i}) = (\theta - \frac{I(q > 0) - F(\theta_i)}{f(\theta_i)})q - \mu q^2$$

Now, this one is

- continuous in type
- has a kink at $q = 0$
- most importantly, it is a "private utility"

So, just like that, we can maximize the sum of virtual (and private) utilities. In other words, we only have to enforce an "efficient" allocation in the virtual economy.

Part 2: Efficient allocation in the virtual economy

There will be two different utilities

- the virtual utility $v(\theta, q)$
- the tax-adjusted utility $u(\theta, q) - \int_0^q m\tau(x, p)dx$

Where the marginal tax is paid per unit dq, at the current clock p, on top of p, presumably in some ascending auction design.

What is the connection?

Now, assuming concavity, the designer wants to set

$$p = mv_i(q, \theta), \quad i = 1, \ldots, n$$

where v_i is the virtual utility.

On the other hand, the agent wants to set

$$mu_i(q, \theta_i) = p + m\tau_i(q, p), \quad i = 1, \ldots, n$$

to maximize his tax-adjusted utility

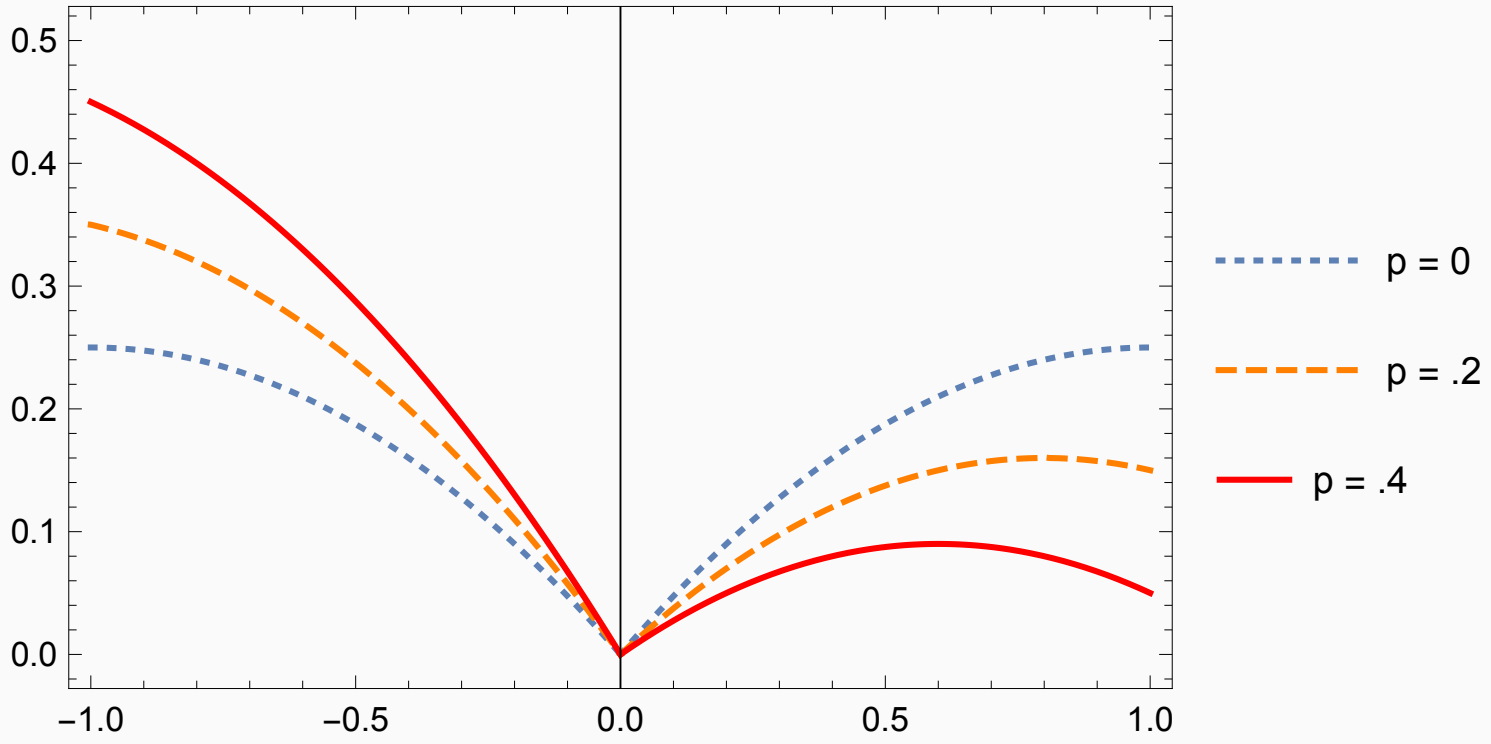
Solve two non-linear equations (separately per agent):

$$p = mu(q, \theta^*) - m\tau(q, p) = mv(q, \theta^*)$$

to eliminate θ^* and recover $m\tau(q, p)$.

This way, sincere bidding agents will (collectively) pick an allocation that is "efficient" in the virtual economy.

Let the utility be quadratic and private type distributed $U[-1, 1]$, for two agents. I can derive the optimal tax (for any distribution, in fact).



Two key features

- kink at zero creates exclusion of weak traders
- shoulders minimize distortion for strong traders

P.S. I did not even solve for the direct mechanism, but I already have a semi-implementation with menus of quantities and prices. Thus, the mechanism (and all the endogenous worst-off-types) are computed by the equilibrium outcomes of the auction like in some human circuit-board.

Part 3: find double-clock implementation

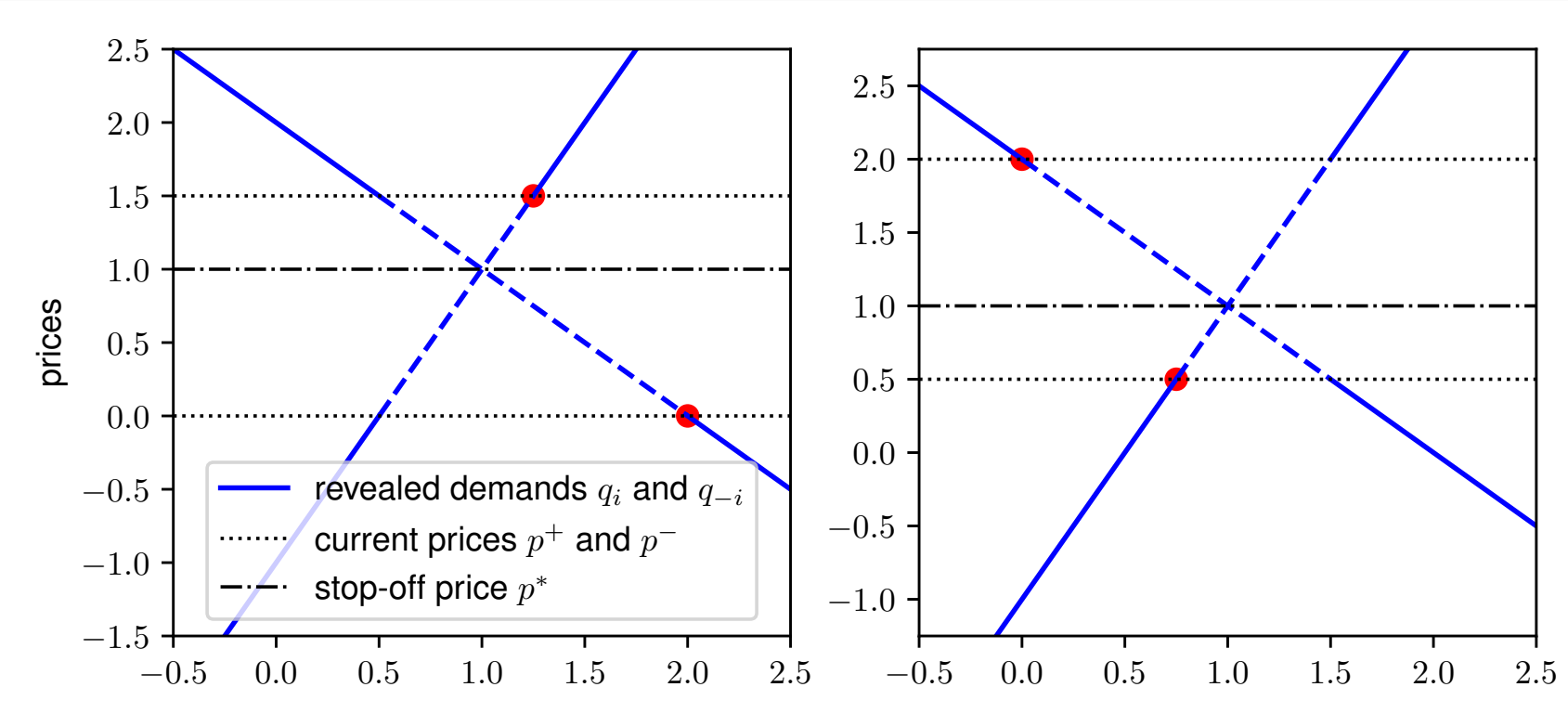
How to design the auction?

- Robustness implies Vickrey-style transfers
- Vickrey + Dynamic = Ausubel (clinching) design
- presumably, two Ausubel auctions running towards each other while continuously clearing the market, until the clock prices meet

That is basically it.

Let $p^+ \leqslant p^-$ be clock prices in forward and reverse auctions.

Let q_i^+ be the revealed demand in forward and $q_i^-(p)$ in the reverse auction. Let $q_{-i}^+ = -\sum_{j \neq i} q_j^+$ the residual demand in forward auction, and $q_{-i}^- = -\sum_{j \neq i} q_j^-$ the residual demand in reverse auction.



Transfers are equal to the area under the "residual" demand.

Part 4: eliminate unwanted equilibria

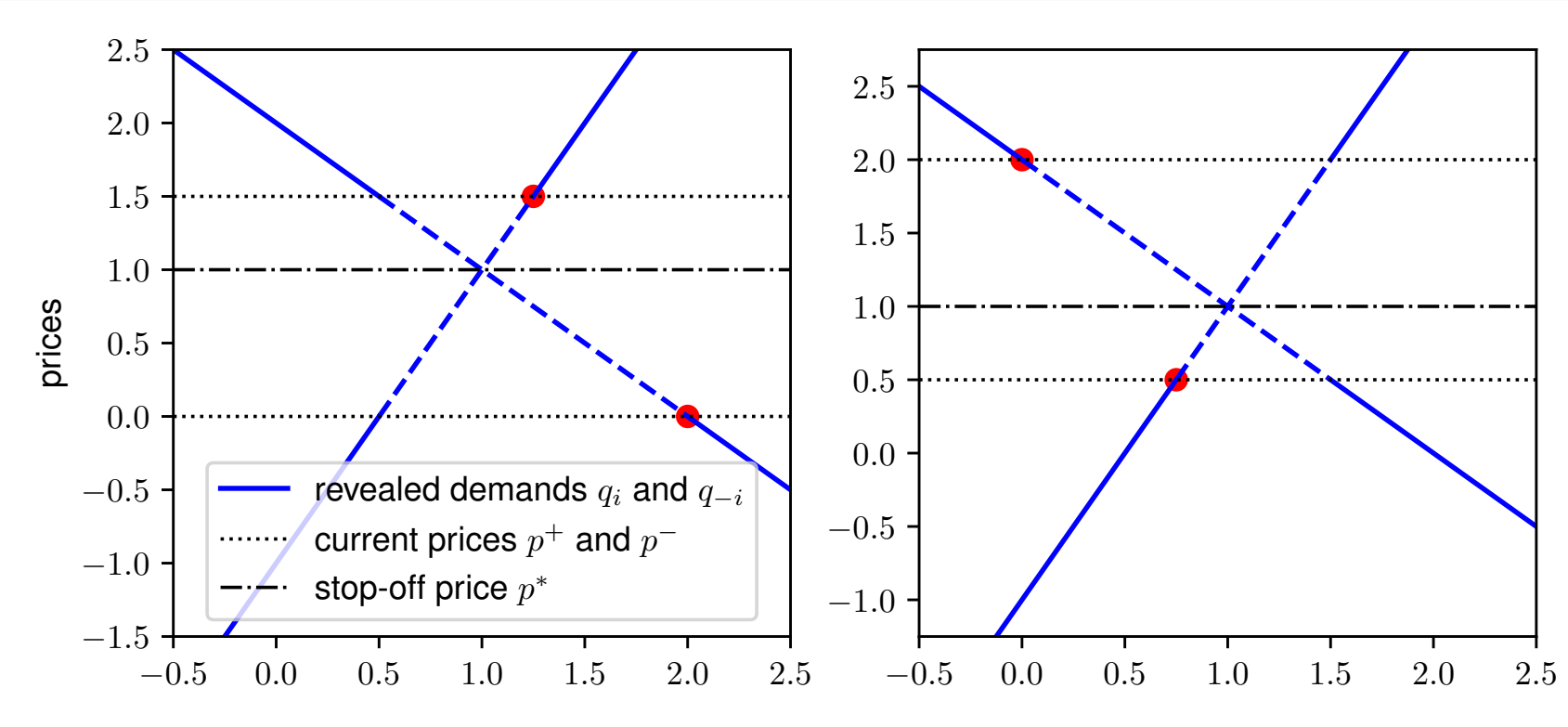
So the implementation is two Ausubel auctions: forward (buyers, +) and reverse (sellers, -), with their price clocks running toward each other.

- the truthful equilibrium exists no matter how you move the clocks
- but the clocks can be moved along different price paths
- this gives us freedom to optimize... or minimize... something

- if we minimize disclosure, there will be 1 equilibrium
- if we maximize disclosure, there will be multiple equilibria

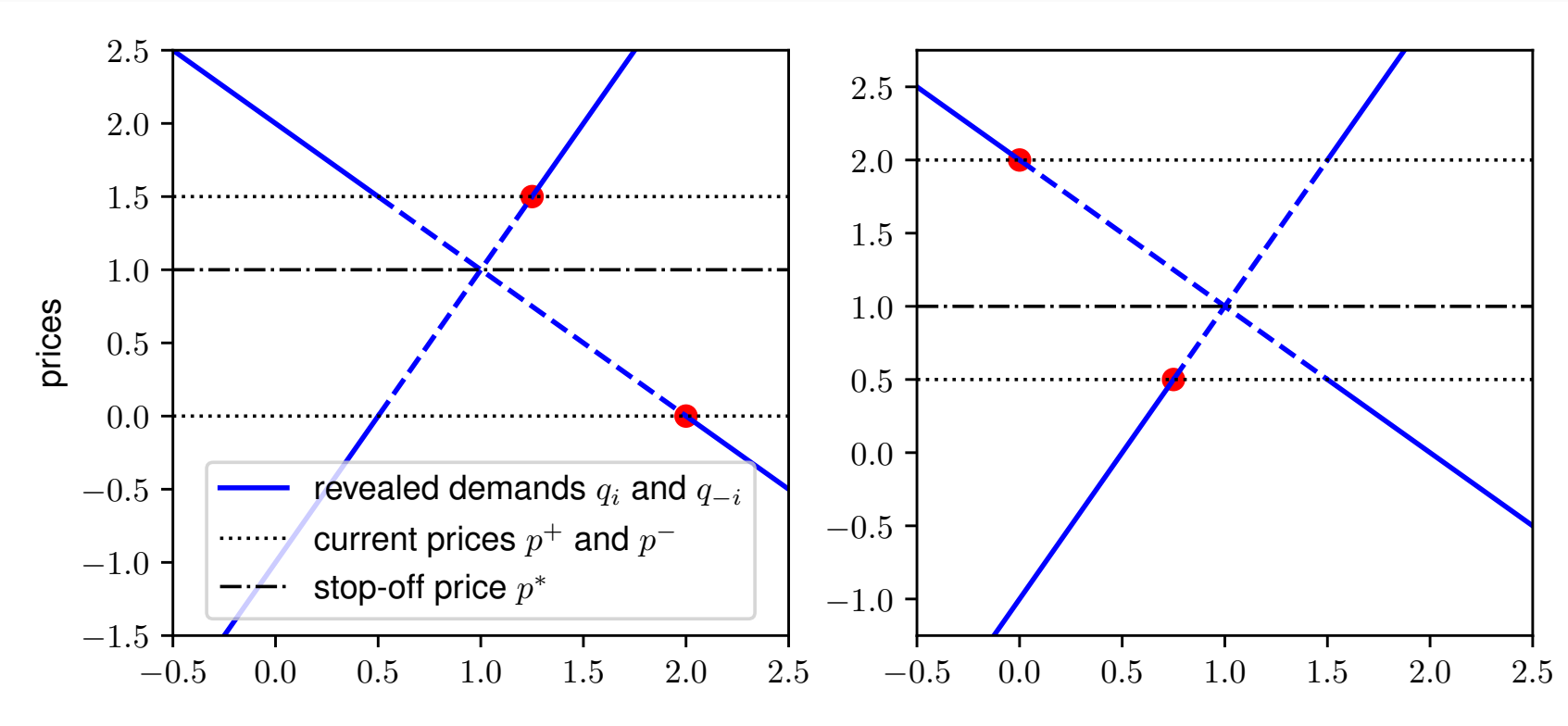
This is an unfortunate consequence of informational spillover between the forward and reverse auctions. As long as bidders participate on both sides or use shills.

Our idea is to minimize unwanted equilibria by moving the clocks in such a way that there is minimal spillover and so there is minimal need to conceal information.



Left figure - spillover into forward auction. Right figure - into reverse.

- Agent i experiences spillover into forward auction iff $q_i^+ > q_{-i}^-$
- Agent i experiences spillover into reverse auction iff $q_i^- < q_{-i}^+$



Left figure - spillover into forward auction. Right figure - into reverse.

- Why do I experience spillover into the forward auction? Because the reverse clock has progressed too much towards the stopoff price.
- Why do I experience spillover into the reverse auction? Because the forward clock has progressed too much towards the stopoff price.

Lemma 2: either there is spillover into only one auction (forward or reverse) or there is spillover for at most one agent.

Proof: assume that there is spillover into both auctions, and also for different agents

- $-\sum_{k\neq i} q_k^+ = q_{-i}^+ > q_i^-$ for some i
- $q_j^+ > q_{-j}^- = -\sum_{k\neq j} q_k^-$ for some $j \neq i$

Then

$$-\sum_{k\neq i,j} q_k^+ > q_i^- + q_j^+ > -\sum_{k\neq i,j} q_k^-$$

or

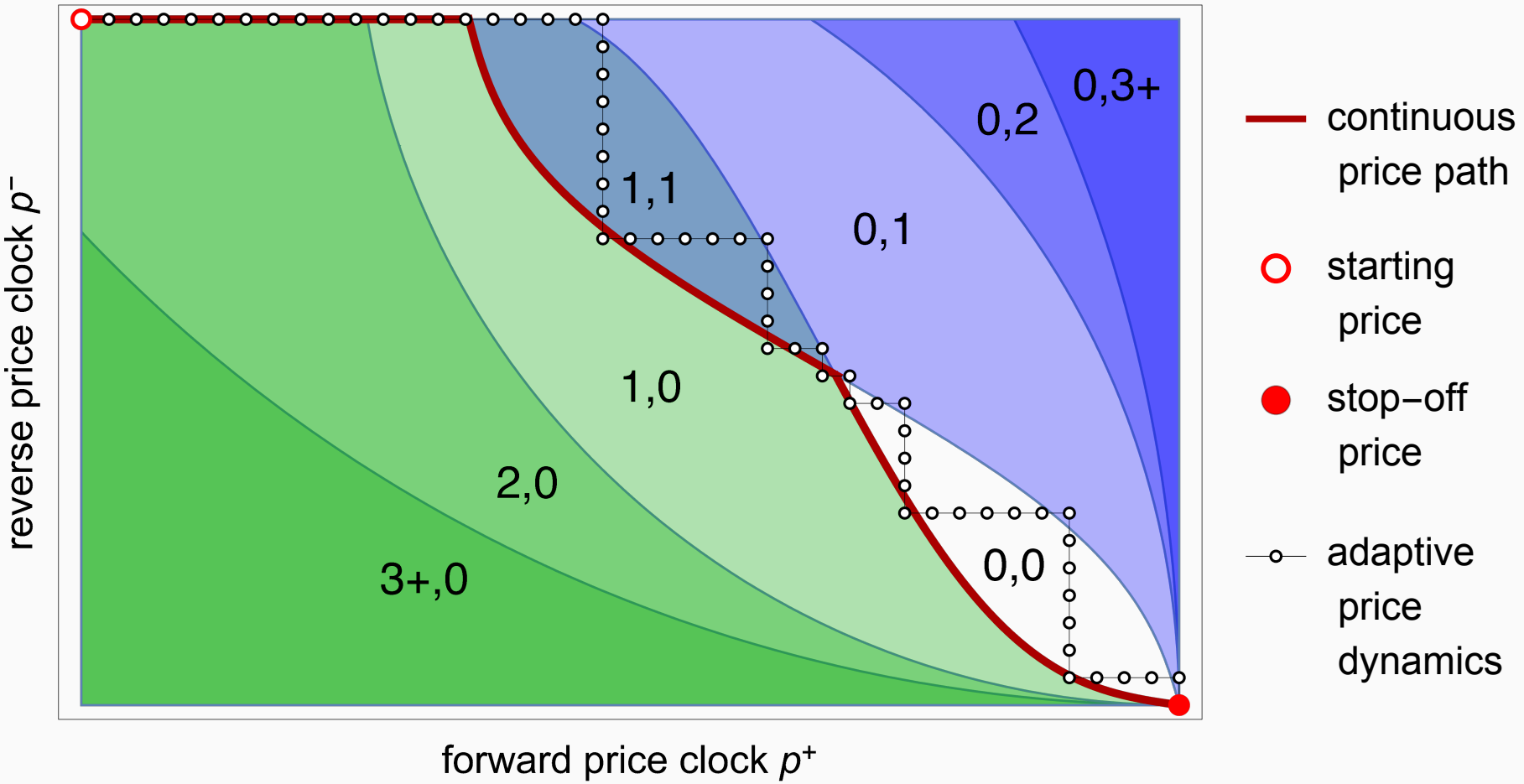
$$\sum_{k\neq i,j} q_k^+ < \sum_{k\neq i,j} q_k^-$$

which contradicts monotonicity of demand ($q_k^- \leqslant q_k^+$).

Adaptive price policy: if there are more spillovers into forward - move forward clock, thus reducing spillovers into forward. If there are more spillovers into reverse - move reverse clock thus reducing spillovers into reverse. Otherwise, move either clock.

Put differently, you can minimize the total number of spillovers by balancing the spillovers into forward and reverse auctions, eventually having only 1 or 0 agents exposed.

We want to reach and maintain this ecosystem for as long as possible.


reverse price clock p^-
forward price clock p^+
0,3+
0,2
1,1
0,1
1,0
2,0
3+,0
0,0
continuous price path
starting price
stop–off price
adaptive price dynamics

Summary

- optimal direct mechanism
- double-clock implementation
- eliminate unwanted equilibria
- minimize spillover (maximize disclosure)
- asymptotic theory (this is new)