Optimal Robust Double Auctions

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Double auctions

	single-unit	multi-unit
one-sided	FPA, English, SPA, Dutch	VCG, GSP, Ausubel
double	McAffee	uniform-price, pay-as-you-bid, Incentive, ???

- uniform-price has 0 budget
- McAffee and Incentive almost break even
- UP, PAYB and Incentive are not robust

There is a missing auction design here, we want to find it.

The closest to what we do are: Wilson (1985), Lu Robert (2001) and Loertsher Wasser (2019), but they are all focused on Bayesian IC, rather than ex-post IC.

This paper

In an environment with many buyers and sellers that have private information and multi-unit demand, but independent utilities...

...we develop a optimal (rev. maximizing) robust (ex-post IC, IR) mechanism and implement it in a dynamic fashion.

Goal 1: find optimal direct mechanism

Goal 2: find double-clock implementation

Goal 3: eliminate unwanted equilibria

However, the order will be a bit strange. We will FIRST find the auction implementation and THEN find the optimal direct mechanism. In a way, the auction will solve for the optimal mechanism, if only we can keep it from producing unwanted equilibria.

Goal 4: optimize over price-clock paths (this is unique to our paper)

Part 1: find optimal robust direct mechanism

Consider a classical single-crossing (but non-linear) consumption utility $u(\theta, q)$ and quasi-linear payoff $u(\theta, q) - t(\theta)$ structure. Let the type θ and allocation q be 1-dim.

Leading example: $u(\theta, q) = \theta q - \mu q^2$, where μ is known

What is an optimal (revenue-maximizing) robust mechanism?

- denote (equilibrium) surplus as $s(\theta) = u(\theta, q(\theta)) - t(q(\theta), \theta_{-i})$
- maximize average $u(\theta, q) - s(\theta)$
- ex post IC constraint $s(\theta) = \max_q [u(\theta, q) - t(q, \theta_{-i})]$
- ex post IR constraint $s(\theta) \geqslant u(\theta_i, 0)$

Standard virtualization techniques apply

$$v(\theta_i, q|\theta_{-i}) = (\theta - \frac{I(\theta_i > wot(\theta_{-i})) - F(\theta_i)}{f(\theta_i)})q - \mu q^2$$

where wot(θ_{-i}) is the worst-off type.

Problems:

- this is not even a "private utility"
- virtual utility is (downwards) discontinuous in own type
- worst-off type is endogenous to the sought mechanism
- worst-off type is conditional on types of others

Seems we have a fixed-point type of problem with ironing on top...

... but don't panic!

Lemma 2: let $\text{tet}(\theta_{-i})$ be the type excluded from trade, that is, who trades exactly zero, then it is one of the worst-off types $\text{wot}(\theta_{-i})$.

$$\text{in other words, } tet(\theta_{-i}) \subset wot(\theta_{-i}).$$

Proof: the slope of surplus is linked to the sign of your trade by single crossing, thus if sub-gradient contains zero it is also the argmin.

This trick only works for the ex-post constraints!!!

For example, the worst-off interim type does not have to trade zero in expectation unless there is extreme symmetry in the model.

Thus virtual utility can be rewritten wlog

$$v(\theta_i, q|\theta_{-i}) = (\theta - \frac{I(\theta_i > tet(\theta_{-i})) - F(\theta_i)}{f(\theta_i)})q - \mu q^2$$

or, put differently

$$v(\theta_i, q|\theta_{-i}) = (\theta - \frac{I(q > 0) - F(\theta_i)}{f(\theta_i)})q - \mu q^2$$

Now, this one is

- continuous in type
- has a kink at $q = 0$
- most importantly, it is a "private utility"

So, instead of facing a crazy hard simultaneous ironing problem, we can maximize the sum of virtual utilities. In other words, we only have to enforce an "efficient" allocation in the virtual economy.

Part 2: find double-clock implementation

How to design the auction?

- Robustness implies Vickrey-style transfers
- Vickrey + Dynamic = Ausubel (clinching) design
- presumably, two Ausubel auctions running towards each other while continuously clearing the market, until the clock prices meet

But how about the budget deficit?

Small detour to Andreyanov Sadzik (2021) paper.

Lets add "quadratic" tax for each transaction, to subsidize the loss associated with Vickrey-style payments.

- marginal tax $m\tau(q) = \sigma q$, where $\sigma > 0$.
- integrated tax $\tau(q) = \sigma q^2/2$

Bidding is sincere in the sense that bidders play truthfully, but as if their utility was deformed $u(q) \to u(q) - \tau(q)$, and the auction finds the efficient allocation in the deformed economy.

Alternatively add a "bid-ask spread"

- marginal tax $m\tau(q) = \delta sign(q)$, where $\delta > 0$.
- integrated tax $\tau(q) = \delta|q|$

Bidding is sincere in the sense that bidders play truthfully, but as if their utility was deformed $u(q) \to u(q) - \tau(q)$, and the auction finds the efficient allocation in the deformed economy.

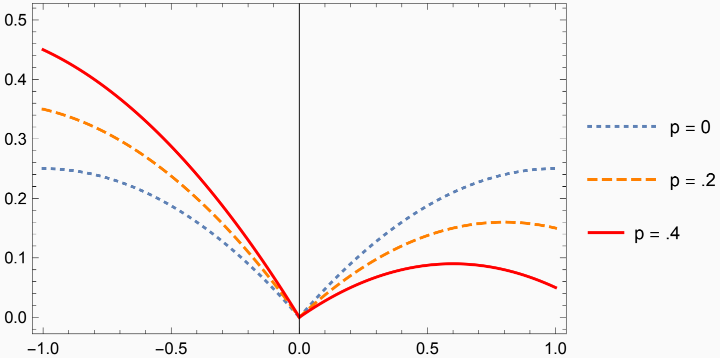
But how should the tax look like for us to find the efficient allocation in the virtual economy?

The optimal (revenue maximizing) tax has to

- depend on the clock price $\tau(p, q)$
- likely regressive (concave) rather than progressive (convex)
- integrated tax $\tau(p, q)$ exhibit a kink at $q = 0$
- equivalently, $m\tau(p, q)$ exhibit a discontinuity at $q = 0$
- does not depend on the number of bidders

Here is an illustration

Let the utility be quadratic and private type distributed $U[-1,1]$, for two agents. I can derive the optimal tax (for any distribution, in fact).



Two key features

- kink at zero creates exclusion of weak traders
- shoulders minimize distortion for strong traders

How to find it?

Solve two non-linear equations:

$$p = mu(q, \theta^*) - m\tau(q, p) = mv(q, \theta^*)$$

to eliminate θ^* and recover $m\tau(q, p)$.

Crucially, I did not even solve for the direct mechanism, but I already have the implementation.

Thus, the mechanism (and all the endogenous worst-off-types) are computed by the equilibrium outcomes of the auction like in some human circuit-board.

Part 3: eliminate unwanted equilibria

So the implementation is two Ausubel auctions: forward (buyers, +) and reverse (sellers, -), with their price clocks running toward each other.

- payments (classical clinching + marginal taxes)
- disclosure policy
- clock policy
- rationing

In Ausubel (2004) two facts were

- with no disclosure, sincere bidding is weakly dominant.
- with full disclosure and full-support beliefs,

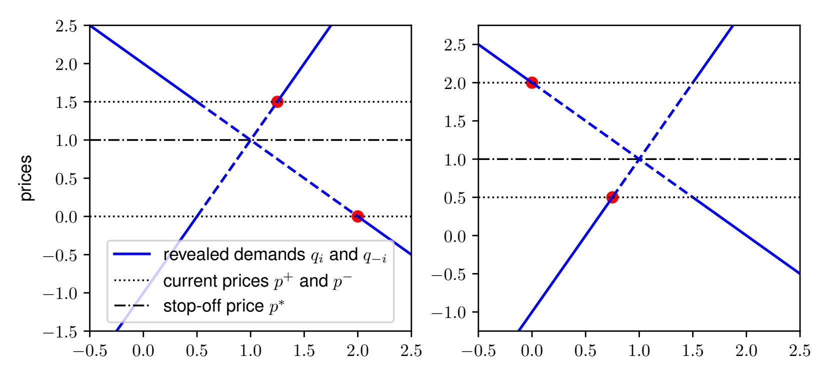
Neither of these is true in the two-sided auction, the reason being

- bidders may learn the stop-off price ahead of time
- bidders may learn if they are buyers/sellers ahead of time

This is a consequence of a more general phenomenon - the inadvertent informational spillover between the forward and reverse auctions.

Let $p^+ \leqslant p^-$ be clock prices in forward and reverse auctions. Let q_i^+ be the revealed demand in forward and $q_i^-(p)$ in the reverse auction.

Let $q_{-i}^+ = -\sum_{j \neq i} q_j^+$ the residual demand in forward auction, and $q_{-i}^- = -\sum_{j \neq i} q_j^-$ the residual demand in reverse auction.



Left figure - spillover into forward auction. Right figure - into reverse.

Let's define it formally

- Agent i experiences spillover into forward auction iff $q_i^+ > q_{-i}^-$
- Agent i experiences spillover into reverse auction iff $q_i^- < q_{-i}^+$

Can we move the clocks to minimize spillovers?

Good news: if we prevent spillover entirely, this will lead to uniqueness (via iterated elimination) of sincere equilibrium, eventually.

Bad news: with 2 players, preventing spillover entirely is not possible.

What is possible then?

Spillover minimization

Lemma 2: either there is spillover into only one auction (forward or reverse) or there is spillover for at most one agent.

Proof: assume that there is spillover into both auctions, and also for different agents

- $-\sum_{k \neq i} q_k^+ = q_{-i}^+ > q_i^-$ for some i
- $q_j^+ > q_{-j}^- = -\sum_{k \neq j} q_k^-$ for some $j \neq i$

Then

$$-\sum_{k \neq i,j} q_k^+ \geqslant q_i^- + q_j^+ \geqslant -\sum_{k \neq i,j} q_k^-$$

or

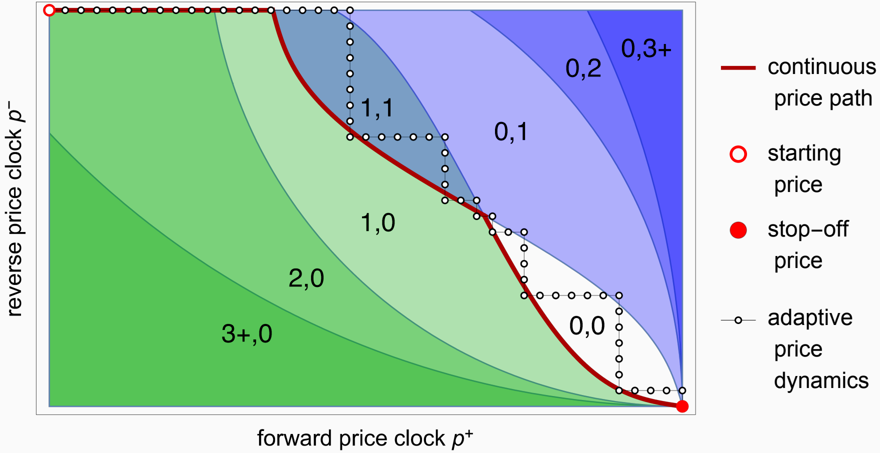
$$\sum_{k \neq i,j} q_k^+ < \sum_{k \neq i,j} q_k^-$$

which contradicts monotonicity of demand $(q_k^- \leqslant q_k^+)$.

Put differently, you can minimize the total number of spillovers by balancing the spillovers into forward and reverse auctions, eventually having only 1 or 0 agents exposed.

We want to reach and maintain this ecosystem for as long as possible.

Adaptive price policy: if there are more spillovers into forward - move forward clock, if there are more spillovers into reverse - move reverse cloc. Otherwise, move either clock.



1,1
0,1
0,2
0,3+
1,0
2,0
3+,0
0,0
reverse price clock p^-
forward price clock p^+
continuous price path
starting price
stop−off price
adaptive price dynamics

Summary

- optimal direct mechanism
- double-clock implementation
- eliminate unwanted equilibria
- minimize spillover (maximize disclosure)